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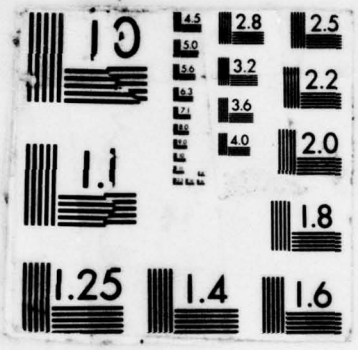
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MODEL STRUCTURE DETERMINATION AND TEST INPUT SELECTION FOR IDENTIFICATION OF NONLINEAR REGIMES

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This report describes techniques for model structure estimation and test input design for system identification in nonlinear vehicle regimes. Optimal methods for parameterizing estimation problems are discussed. For those regions where aerodynamic or hydrodynamic force and moment models are not known, the use of splines in approximating unknown nonlinear effects is discussed. Finally, methods for selecting inputs which enhance system identifiability are described.

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CHAPTER I

INTRODUCTION

The significant improvement in Navy parameter identification capability has been demonstrated in recent aircraft flight test programs. The utilization of this capability has brought the parameter identification algorithms to a point where restrictions to the applications of this technology occur because of imprecisely defined a priori model forms and poorly excited modes [1-3].

The problem of ill-defined model structures occurs naturally as the application of the system identification technology is extended to more complex Navy systems (e.g., surface and subsurface marine vehicles, aircraft, inertial navigation components). Additional complexity arises in those operation regimes which are characterized by significant variation in governing physical phenomenon (e.g., separation of flow at high angle-of-attack). This problem occurs because of two main reasons: (1) these systems often cannot be well analyzed by theoretical methods and the assumptions of the analyses are questionable in extreme regimes, and (2) even if a reasonable theoretical investigation could be carried out, the resulting model may be too complex to be useful for engineering or simulation objectives. To complement theoretical models, it is therefore desirable to have techniques for estimating the structure of linear and nonlinear models, of varying complexity, from test data. Such model structure determination algorithms, when applied to a set of data, could give the simplest model which substantiates the data. The resulting models may be used, for example, in control system design, handling qualities evaluation, stability analysis, and simulation verification.

Proper excitation of significant modes is necessary to effectively estimate both model structure and the parameters of that

model structure. Specification of the test procedure is, thus, of paramount importance in high order systems and in complex nonlinear systems. When there are many important modes, a conventional input (e.g., step, pulse) may not excite all the modes leading to identifiability problems. In addition, in nonlinear systems, it may not be feasible to "hold" the system in any particular regime for a sufficient time to acquire enough data, because of safety and stability considerations. Therefore, inputs which are effective in giving good identifiability of parameters under amplitude or time constraints are important for practical identification applications.

This report describes the work in areas of model structure determination and input selection. Special emphasis is given to their application to nonlinear systems.

1.1 PRINCIPAL DEVELOPMENTS

Thw work conducted under the present effort has produced many theoretical and algorithmic developments. The main results are summarized here.

The following theoretical results are obtained:

(1) Optimal Selection of Parameterization in Estimation Problems: Methods, based upon the likelihood function, have been developed for selection of a set of parameters which best describe the observed data. Extraneous parameters are dropped.

(2) Model Structure Estimation Using Splines: Use of polynomial splines in representing unknown nonlinearity and time variation is investigated. System identification methods based upon such representation are presented.

(3) Input Design for Model Discrimination: Inputs, which enhance differentiability among many models based upon data, are studied and methods to select such inputs are investigated.

In addition to these theoretical results, several results are obtained which should improve current system identification algorithms. These include:

(1) Improved Techniques for Isolation of Poorly Identifiable Parameters: The parameters, which are only marginally identifiable, can be discarded with improved reliability with the development of better parameterization techniques.

(2) Optimal Subset Regression on Spline Representation: Method for determining a useful spline representation through the application of the optimal subset regression technique have been developed.

(3) Suboptimal Multistep Input: A practical technique for designing suboptimal multistep inputs for linear and nonlinear systems has been developed.

Both the theoretical and the algorithmic developments have advanced state-of-the-art of system identification, in particular, its application to the formulation of mathematical models for Navy vehicles.

1.2 SUMMARY

This report is organized as follows. Chapter II discusses, in brief, the previous results in the areas of model structure determination and input design. New model structure estimation methods are developed in Chapter III, followed by the input design techniques in Chapter IV. Chapter V gives the conclusions. Applications of these algorithms to testing of Navy vehicles and other systems is described in other reports and technical papers.

CHAPTER II

REVIEW OF PREVIOUS WORK

2.1 INTRODUCTION

The problem of model structure estimation and the importance of selecting appropriate test signals have long been recognized. Only preliminary work, however, has been done on these problems because even the basic techniques for parameter estimation were unavailable until recently. Most of the work on model structure determination and input designs until now has been applicable only to linear systems. Previous model structure determination and input design methods are now discussed.

2.2 TECHNIQUES FOR MODEL STRUCTURE DETERMINATION

Methods for isolating a model structure from the response data have been developed mainly for linear systems. In linear systems, the problem of model structure estimation is simpler because the structures of multiinput/multioutput linear systems is completely defined by a finite set of real integers called the canonical indices [4-6].

The current techniques for finding the canonical indices from measured data use various approaches. Tse and Weinert [7] use the fact that Hankel matrices of orders higher than a certain value are singular. They compute the highest order of the Hankel matrix whose determinant is nonzero. This gives one of the canonical indices. This process is repeated for all canonical indices. Such a method may work well if the number of inputs and outputs is small and the number of data points is large. Often

however, it is unsatisfactory because statistical properties of the determinant of the Hankel matrix are not known.

Akaike [8] has also developed a method for determining model structures of linear systems. The method selects canonical indices based upon the minimization of the prediction error of the outputs (a criterion called the final prediction error criterion is used). It is shown, that in the stochastic case, the dimension of this predictor space is equivalent to the system order. This method has been applied quite extensively with some success [9] and some problems [10]. Modifications to solve some of the problems with the Akaike final prediction error criterion have been suggested [10]. Similar criteria have been suggested by Parzen [11] and others. Chi-square and statistical F-ratio tests may also be applied.

Estimation of model structures in nonlinear systems is a relatively newer field and few results are available. The pioneering work in this area was done by Hall and Gupta [12]. They used a general polynomial representation for the unknown nonlinear relationships and an optimal subset regression approach to estimate the unknown nonlinear aerodynamic effects at high angle-of-attack. The results were applied to flight data from an F-4 aircraft with good success. This procedure, however, has limitations. The polynomials may be inadequate for approximating some nonlinearities commonly encountered in physical systems. In addition, polynomial models may be computationally marginally stable. Also, the coefficients of the polynomial terms may have an unacceptably large correlation. It is apparent, therefore, that the parameters have to be carefully selected in nonlinear estimation problems, resulting in a need to use a better representation for the nonlinearities.

2.3 TECHNIQUES FOR INPUT DESIGN

Techniques for input design have been developed only for the linear systems. The selection of test signals was first considered by Goodwin [13], Reid [14], and Mehra [15]. Goodwin and Reid worked with the parameter covariance matrix and tried to optimize it directly. The optimization problem was so complex that only very simple problems could be solved. Mehra [15] worked with the trace of the information matrix which gave a simpler optimization problem, but the inputs were not very useful. Later, Gupta and Hall [16], Mehra [17] and Mehra and Gupta [18] were able to simplify the procedure for optimizing various functions of the covariance matrix. This made it possible to solve fairly complex problems, but the computation time was still quite high. Recently, G. Reid [19] has shown how sums of Walsh functions may be used to further reduce the computation time.

2.4 SUMMARY

Past work on input design and model structure estimation has been, in a great part, limited to linear systems and simple criteria. There is a need to extend these methods to produce new algorithms for nonlinear systems. In addition, the techniques for linear systems need to be futher developed so that they are computationally feasible for modestly complex systems.

CHAPTER III

MODEL STRUCTURE DETERMINATION TECHNIQUES

3.1 INTRODUCTION

The development of a useful mathematical model from a given response data involves two major steps: (1) model structure estimation, and (2) parameter identification. The parameter identification problem has been a subject of research of many authors leading to several techniques which can be applied to linear, as well as nonlinear, systems [20-22].

The model structure estimation problem has to be treated separately for linear and nonlinear systems. In linear systems, this problem simplifies considerably because the basic structure of linear models is completely specified by a finite set of real integers called canonical indices. Therefore, evaluation of this small set of real integers completely determines the structure of linear models. Techniques for estimating canonical indices from response data have been developed by Akaike [8], Tse and Weinert [7] and Vanden Boom, et al. [23], as discussed in the previous chapter.

Absence of such convenient indices in nonlinear systems makes the problem of determining model structure particularly difficult. It is to be mentioned that the estimation of a completely unknown nonlinear function from data contaminated with noise is not a well-posed problem [24]. Therefore, the best that can be hoped is to get a good approximation for the nonlinearities.

During the course of the present effort, two methods have been developed. The first method starts off with the most complex model of the system which may be expected from physical or other

a priori considerations. Then the terms which are not relevant in describing the response are dropped. As we show in Section 3.2, this technique is useful in accurate estimation of parameters of interest from finite data, when the model also contains other parameters, whose values are not of any direct interest.

The second method utilizes splines to approximately model unknown, nonlinear functional relationships and time variations in the representation of a dynamic system. A set of important terms is selected by the application of the subset regression technique.

Section 3.2 gives the theoretical background of the method of the selection of an efficient parameterization for a response data. Some simple examples are presented and its relationship to other methods is illustrated. Section 3.3 discusses the application of polynomial splines in the modeling of nonlinear systems of unknown structure. The method is demonstrated by using examples of simulated data and flight test data.

3.2 CHOICE OF PARAMETERIZATION IN ESTIMATION PROBLEMS

Most physical processes are governed by a complex interaction of forces. An exact modeling of such processes is neither feasible nor desirable. In general, a complex model may be divided into two parts: a part which is of direct interest for any application (called the primary portion) and a part which is not of any direct interest (called the secondary portion). The two portions of the model, however, are not independent because of cross-coupling terms. Because of this coupling, certain parts of the secondary portion must be included even though the basic emphasis is on identifying the primary portion. Often the entire secondary portion may not be included because: (a) this will provide poor estimates of the primary parameters by scattering a finite information over too many parameters, and (b) there may be numerical

problems during estimation. Therefore, useful parameterization must be selected such that the primary parameters are estimated with maximum accuracy and the model is manageable. It should be noted that dropping some of the secondary parameters will give biased estimates of the primary parameters. These biased estimates, however, have a lower mean-square estimation error if the parameterization is selected carefully.

We begin with two examples in Section 3.2.1. General nonlinear systems are considered in Section 3.2.2. Section 3.2.3 gives the summary and the application of the method to the general model structure determination problems.

3.2.1 Two Examples

To motivate the results in the next sections, two examples are now given.

Example 3.1

Consider a nonlinear regression example in which the input u and the output y are related by the equation

$$y = au + bu^2 + v \tag{3.1}$$

v is random noise with unit variance. This is the true model. Suppose, we are interested in determining parameter 'a' accurately. Then the primary model is

$$y = au + v' \tag{3.2}$$

Several input/output measurements at (u_i, y_i), $i = 1,2, \ldots, r$ are taken. Then if the true model is used the parameter estimates and the covariance of estimation errors are

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} R_{uu} & R_{uu^2} \\ R_{u^2u} & R_{u^2u^2} \end{bmatrix}^{-1} \begin{bmatrix} R_{uy} \\ R_{u^2y} \end{bmatrix} \tag{3.3}$$

$$\text{cov} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} R_{u^2u^2} & -R_{u^2u} \\ -R_{uu^2} & R_{uu} \end{bmatrix} \tag{3.4}$$

where

$$R_{xy} = \sum_{i=1}^{r} x_i y_i \tag{3.5}$$

is the cross-correlation coefficient, and

$$\Delta = R_{uu} R_{u^2u^2} - (R_{uu^2})^2 \tag{3.6}$$

If the primary model is used, the estimate of 'a' is

$$\hat{a}_b = R_{uy}/R_{uu} \tag{3.7}$$

The variance of this estimate is

$$\text{Var } (\hat{a}_b) = \frac{1}{R_{uu}} \tag{3.8}$$

However, this estimate is biased. The bias in the estimate is

$$\text{Bias } (\hat{a}_b) = \frac{aR_{uu} + bR_{u^2u}}{R_{uu}} - a = \frac{bR_{u^2u}}{R_{uu}} \tag{3.9}$$

The mean-square error of this estimate is

$$\text{MSE } (\hat{a}_b) = \frac{1}{R_{uu}} + \frac{b^2(R_{u^2u})^2}{(R_{uu})^2} \tag{3.10}$$

Case (1): Suppose that a is to be estimated from two measurements at $u = 1,2$. Figure 3.1 shows the mean-square error of estimates of Eqs. (3.3) and (3.7) (solid line). Note that it is better to use the simplified model when $b < 1.12$.

Case (2): Suppose now that a new measurement is added at $u = 3$. The broken lines show the variation of mean-square error with b. Again for $b < 0.441$, it is better to use the simplified model than the true model.

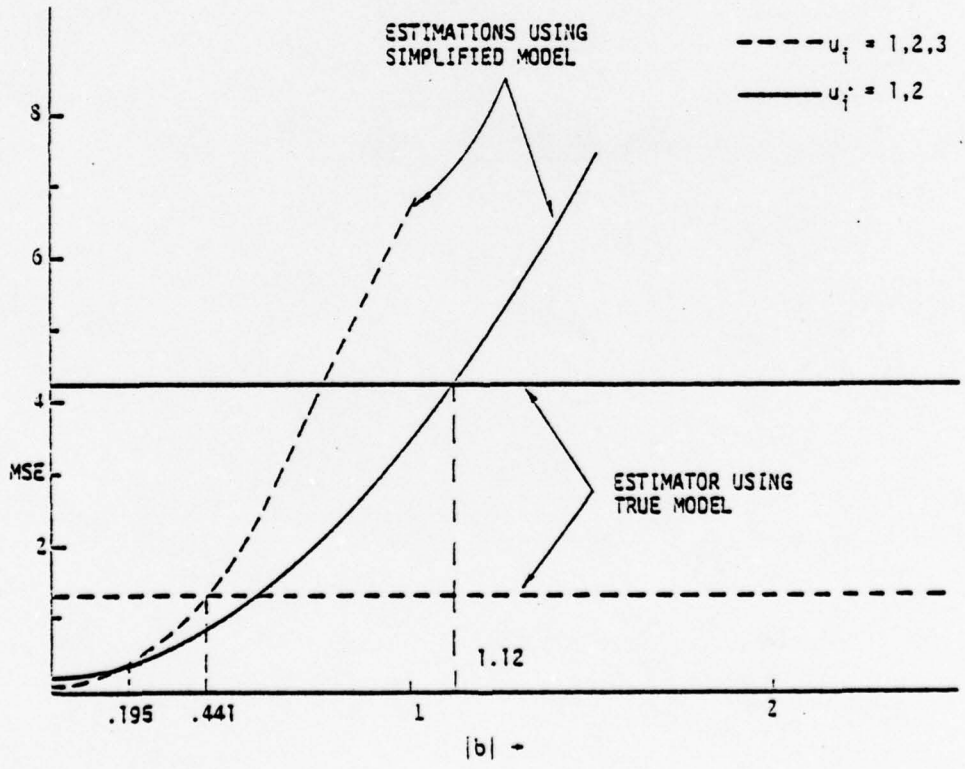

Figure 3.1 Mean-Square Errors of Estimate of Eqs. (3.3) and (3.7)

The most interesting result is obtained for $0.195 < b < 0.58$. For this range of b values, using the measurements at two inputs $u_i = 1,2$ with the simplified model gives lower mean-square error than the case where all three measurements are used either with the true or the simplified model. In other words, for the purpose of estimating a, the third measurement should be rejected. This is an example where the measurement contains information, but cannot be used for estimation.

Example 3.2

Consider now a dynamic system

$$\dot{x} = -ax + u + b, \qquad x(0) = 0 \tag{3.11}$$

a is the time constant and the parameter of interest. b is a bias from uncertain initial condition. Let there be noisy measurements of the state

$$y = x + v \tag{3.12}$$

v is white random noise with power spectral density of 0.01. Let,

$$u = 1 \quad 0 < t < 1 \tag{3.13}$$

Then

$$x = \frac{1 + b}{a} (1 - e^{-at}) \tag{3.14}$$

If the parameter b is neglected the estimator a_b will be biased. Its variance, bias and mean-square error are

$$\text{Var}(\hat{a}_b) = \frac{0.029}{(1+b)^2}$$

$$\text{Bias}\ (\hat{a}_b) = \frac{0.626b}{(1+b)}$$

$$\text{MSE}(\hat{a}_b) = \frac{0.392b^2 + 0.029}{(1+b)^2} \qquad (3.16)$$

The root-mean-square error as a function of b is shown in Figure 3.2. Thus b should be disregarded, if it is known to lie between $\pm$ 0.55, but its exact value is not known.

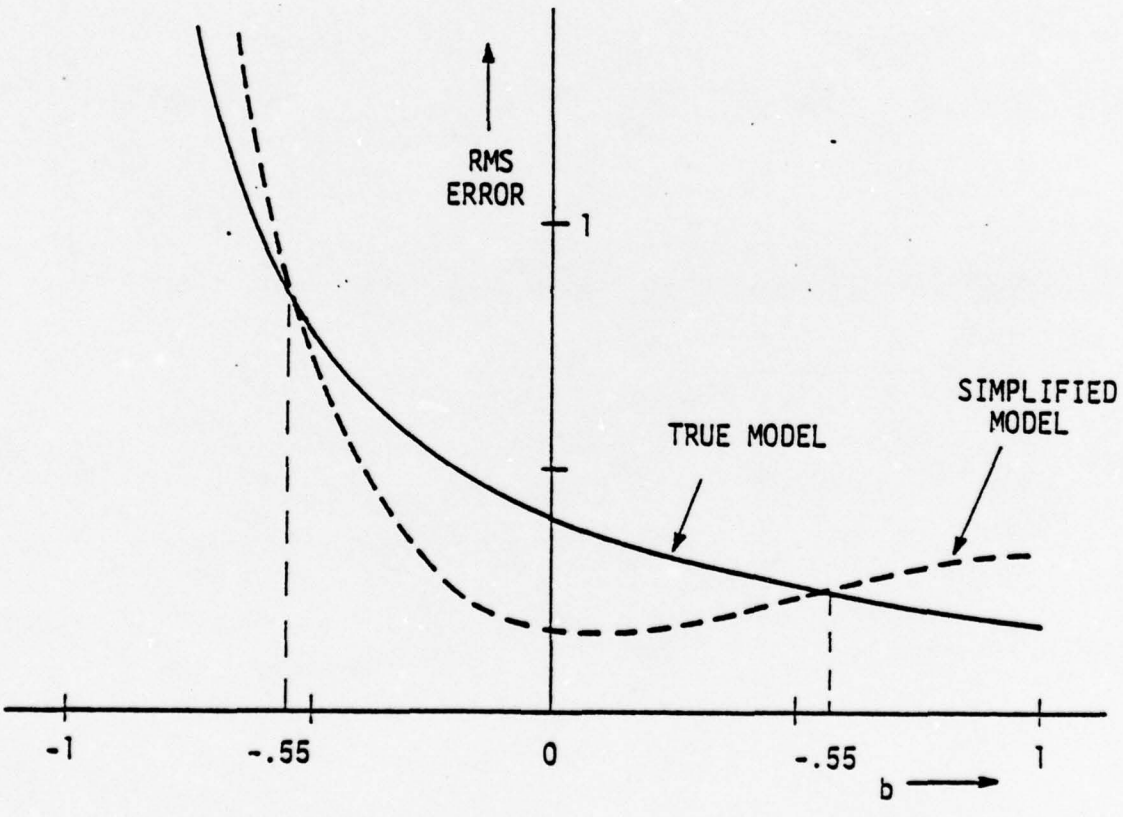

Figure 3.2 Variation of the RMS Error with b for the Estimators Based Upon the True and the Simplified Model

3.2.2 Choice of Parameterization in General Dynamic Systems

Consider a continuous time system, in which the $n \times 1$ state x and the $p \times 1$ output y follow the equations

$$\dot{x} = f(x,u,t,\theta,\varphi) + w(t) \tag{3.17}$$

$$y = h(x,u,t,\theta,\varphi) + v(t) \tag{3.18}$$

where u is the $q \times 1$ vector of the deterministic inputs and $w(t)$ and $v(t)$ are random noises. The state equation and measurement equation are parameterized on two sets of parameters. θ is the $m_1 \times 1$ vector of parameters whose value we are interested in estimating. The value of φ is not known and is not of direct interest. Nevertheless, because of its effect on the equations of motion, φ must be given due consideration. Examples of φ are bias in instruments, initial condition errors, parameters governing coupling to other approximately independent modes, etc.

There are several important questions which come up at this point. Can the parameters φ be neglected? Should some or all of them be estimated together with θ? Under what circumstances is it better to neglect φ than to estimate φ? The technique for selecting useful parametrization would answer these questions.

Let $J(\theta,\varphi)$ be the negative log-likelihood function with the global minima at θ_m,φ_m. If the parameters φ are estimated together with θ, the resulting values are θ_m and φ_m. These values are asymptotically unbiased and the covariance of the estimates θ_m and φ_m is

$$\operatorname{cov}\begin{bmatrix} \theta_m \\ \hline \varphi_m \end{bmatrix} = M^{-1} \tag{3.19}$$

$$M = \left[\begin{array}{c|c} \dfrac{\partial^2 J}{\partial \theta^2} & \dfrac{\partial^2 J}{\partial \varphi \partial \theta} \\ \hline \dfrac{\partial^2 J}{\partial \theta \partial \varphi} & \dfrac{\partial^2 J}{\partial \varphi^2} \end{array}\right]_{\substack{\theta=\theta_m \\ \varphi=\varphi_m}} \triangleq \left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right] \tag{3.20}$$

The covariance of θ_m can be written explicitly

$$\text{cov}(\theta_m) = M_{11}^{-1} + M_{11}^{-1}M_{12}[M_{22} - M_{21}M_{11}^{-1}M_{12}]^{-1} M_{21}M_{11}^{-1} \tag{3.21}$$

Suppose that we estimate only the required parameters θ and disregard the parameters φ. Quite clearly, the estimates of θ, say θ_r, will be biased. If φ are set to zero, the estimates θ_r will be a solution of m_1 equations

$$\frac{\partial J}{\partial \theta}(\theta_r, 0) = 0 \tag{3.22}$$

Expanding the above equation around the corresponding unbiased estimate $\bar{\theta}$ of θ_r, we get,

$$\frac{\partial J}{\partial \theta}(\theta_r, 0) = \frac{\partial J(\bar{\theta}, \varphi)}{\partial \theta} - \frac{\partial^2 J(\bar{\theta}, \varphi)}{\partial \varphi \partial \theta}\varphi + \frac{\partial^2 J(\bar{\theta}, \varphi)}{\partial \theta^2}(\theta_r - \bar{\theta})$$

$$+ \text{Higher Order Terms} \tag{3.23}$$

$$E(\theta_r - \bar{\theta}) \cong \left\{\frac{\partial^2 J(\bar{\theta}, \varphi)}{\partial \theta^2}\right\}^{-1} \frac{\partial^2 J(\bar{\theta}, \varphi)}{\partial \varphi \partial \theta}\varphi \tag{3.24}$$

Assuming now that the second derivatives of J are approximately the same at $(\bar{\theta},\varphi)$ as at (θ_m,φ_m), we get

$$\text{Bias } (\theta_r) \cong M_{11}^{-1} M_{12}\varphi \tag{3.25}$$

The covariance of the estimated θ_r is

$$\text{cov } (\theta_r) = M_{11}^{-1} \tag{3.26}$$

The mean-square error of the estimate θ_r is

$$\text{MSE } (\theta_r) = M_{11}^{-1} + M_{11}^{-1} M_{12}\, \varphi\varphi^T\, M_{21}M_{11}^{-1} \tag{3.27}$$

Since θ_m is unbiased, its MSE is equal to its covariance of Eq. (3.21). It is clear that

$$\text{MSE } (\theta_r) \lesseqgtr \text{MSE } (\theta_m)$$

if

$$E(\varphi\varphi^T) \lesseqgtr (M_{22} - M_{21}M_{11}^{-1}M_{12})^{-1} \tag{3.28}$$

where the second inequality implies the positive-definiteness of the difference. Often, the difference between $E(\varphi\varphi^T)$ and $(M_{22} - M_{21}M_{11}^{-1}M_{12})^{-1}$ is indefinite. Tests must be made on individual elements of φ to determine which of them should be included in the identified model and which ones disregarded. $(M_{22} - M_{21}M_{11}^{-1}M_{12})^{-1}$ is the covariance of the estimation errors in φ, if it is estimated.

This procedure for including parameters in φ for identification requires a priori information about the possible variations in the parameters φ. This value of φ is not intended to supply any information about the parameters, but is meant solely to

direct us towards selection of useful parameterization in the estimation of θ.

What if the range of variations of φ is not known? Since φ should be smaller than the standard deviation of its estimation error, it is not possible to use the estimated value of φ to check the inequality of Eq. (3.28) directly. An alternate procedure is proposed here.

Let $J(\theta_r, 0)$ be the minimum value of the negative log-likelihood function when φ is excluded from identification. If at this point φ is included in the minimization, the step in θ and φ, for small φ, would be

$$\begin{bmatrix} \Delta\theta \\ \hline \Delta\varphi \end{bmatrix} = M^{-1} \begin{bmatrix} 0 \\ \hline \frac{\partial J}{\partial \varphi} \end{bmatrix}_{\substack{\theta = \theta_r \\ \varphi = 0}} \tag{3.29}$$

It is straightforward to show that the decrease in the likelihood function will be

$$\Delta J = -\frac{1}{2} \left(\frac{\partial J}{\partial \varphi}\right)^T (M_{22} - M_{21} M_{11}^{-1} M_{12})^{-1} \frac{\partial J}{\partial \varphi} \tag{3.30}$$

ΔJ may be computed numerically. It can be shown that $(-2\,\Delta J)$ has the distribution $\chi^2(m_2)$ under the hypothesis that φ is zero [1,2]. If the reduction in J is significant, φ or the elements φ under consideration must be included in the identification model. For a 95% confidence level, one or more elements of φ must be included in the identified parameters, if,

$$(-2\,\Delta J) > C \quad : \quad P(\chi^2(m_2) < C) = 0.95 \tag{3.31}$$

Note that C does not depend upon the order of θ or the number of data points, but only on the confidence level and m_2.

$\frac{\partial J}{\partial \varphi}$ in Eq. (3.30) consists of two parts: the stochastic part, because of random noise in the system, and the deterministic part because of errors in θ and φ. Using the fact,

$$E\left\{\left(\frac{\partial J}{\partial \varphi}\right)_{\text{random}} \left(\frac{\partial J}{\partial \varphi}\right)^T_{\text{random}}\right\} = M_{22} \tag{3.32}$$

it is easy to show that

$$E(\Delta J) = -\frac{m_2}{2} - \text{tr}\left\{(M_{22} - M_{21}M_{11}^{-1}M_{12})\,\varphi\varphi^T\right\} \tag{3.33}$$

Consider the case where parameters in φ are treated sequentially. Then, at any point in the recursive procedure $m_2 = 1$ and φ is a scalar. So,

$$E(\Delta J) = -\frac{1}{2} - \frac{1}{2}(M_{22} - M_{21}M_{11}^{-1}M_{12})\varphi^2 \tag{3.34}$$

If we choose the 95% confidence interval as the criterion for including the set of identified parameters

$$P(\chi^2(1) < 3.84) = 0.95 \tag{3.35}$$

Therefore, $E(\Delta J)$ will meet the above criterion for including φ if

$$|\varphi| > \frac{\sqrt{2.84}}{(M_{22} - M_{21}M_{11}^{-1}M_{12})^{\frac{1}{2}}}$$

$$= \frac{1.68}{(M_{22} - M_{21}M_{11}^{-1}M_{12})^{\frac{1}{2}}} \tag{3.36}$$

This criterion, in the expected value sense, comes close to that of Eq. (3.28).

3.2.3 Summary

The last section presented a technique for selecting useful parameterization in parameter estimation problems. The properties of the likelihood function near its optimum value were applied and a quantitative measure was developed for inclusion of parameters, whose value is of no direct interest. In stability and control derivative extraction, these parameters usually include initial conditions, bias and other errors in instruments. This procedure, when applied to dynamic systems, is the complement of the F-test of linear step-wise regression.

3.3 MODELING OF NONLINEAR AND TIME VARYING SYSTEMS USING SPLINE REPRESENTATION

In many systems, the physical processes are so complex, that it is not possible to parameterize the effects of various forces from a priori analysis. An example is the dynamics of a high performance fighter airplane in the high angle-of-attack flight regime. This section develops a technique for the determination of a representation of the model of a nonlinear or time varying system with unknown structure and the identification of the parameters in the representation using response data. First, it is necessary to select a representation which is general enough to include all possible models within the a priori constraints. Multivariable splines are used for this purpose. A particular form, which best describes the response data, is then selected. Finally, the parameters of this reduced form are identified by using the maximum likelihood or another method.

Section 3.3.1 discusses splines. In Section 3.3.2 we show how an unknown static function may be approximated by splines when the values of the function contaminated by noise are given at discrete points. Section 3.3.3 shows how nonlinear systems may be represented in terms of the splines. In Section 3.3.4, we consider as a special case the problem of approximating the control nonlinearity in a first order system. The general problem of multiinput/multioutput systems with multivariable nonlinearities is disucssed in Section 3.3.5. Section 3.3.6 gives an example of the identification of nonlinear aerodynamic derivatives of a vehicle. The results are summarized in Section 3.3.7.

3.3.1 Polynomial Splines

Polynomial splines are piecewise polynomials in one or more variables. Splines in single variables have been studied extensively [25] and have been applied to data smoothing and approximation of known functions [26,27]. Study of multivariable splines is quite recent and somewhat incomplete [28]. In this work, we use only polynomial splines; therefore, the word spline will be used to mean polynomial spline. A definition of polynomial spline in a single variable is now given.

Definition 3.1

A function, $S_{m,\nu}(x)$, is a polynomial spline function of order m and degree of continuity ν $(m > \nu)$ in $x \varepsilon (a,b)$ if, and only if:*

*Usually polynomial splines are defined with $\nu = m-1$. We deviate from this conventional definition so that it is possible to get better approximations for functions $f(x)$ which are continuous, but do not have continuous derivatives of high order. Sometimes such splines are said to have each knot of multiplicity $m - \nu$.

(1) There exists an increasing sequence of real numbers $a = x_1 < x_2 < \ldots < x_{k+1} = b$ such that $S_{m,\nu}(x)$ is a polynomial of degree m or less in x between (x_i, x_{i+1}), $i = 1,2,\ldots k$.

(2) $S_{m,\nu}(x)\ \varepsilon C^{\nu}(a,b)$, where $C^{\nu}(a,b)$ is the class of functions continuous through the νth differentiation over the interval (a,b).

Definition 3.2

$x_1, x_2, \ldots, x_k$ are called the knots of the spline function.

Example 3.3

The following function is considered

$$S_{3,1}(x) = \begin{cases} x^3 - 4x^2 + 2 & -2 \leq x < 0 \\ 5x^2 + 2 & 0 \leq x < 1 \\ -x^3 + 8x^2 - 3x + 3 & 1 \leq x < 2 \end{cases} \qquad (3.37)$$

The first and second derivatives of this spline function are

$$S'_{3,1}(x) = \begin{cases} 3x^2 - 8x & -2 \leq x < 0 \\ 10x & 0 \leq x < 1 \\ -3x^2 + 16x - 3 & 1 \leq x < 2 \end{cases} \qquad (3.38)$$

and

$$S''_{3,1}(x) = \begin{cases} 6x - 8 & -2 \leq x < 0 \\ 10 & 0 \leq x < 1 \\ -6x + 16 & 1 \leq x < 2 \end{cases} \qquad (3.39)$$

Note that $S'_{3,1}(x)$ is continuous everywhere, whereas $S''_{3,1}(x)$ is continuous at $x = 1$, but discontinuous at $x = 0$.

The parameters of a spline in a single variable are:

(1) the order of the polynomial, m and the degree of continuity, ν;

(2) the number and positions of knots; and

(3) the coefficients of the polynomial terms.

In the identification of system model from noisy data, it is important to select neither too many, nor too few, knots. If there are many more knots than are reasonable, the spline function is unnecessarily complex and the estimates of knot positions and polynomial coefficients have large errors. Too few knots may not be adequate to approximate a nonlinear function. In addition to the number of knots, the position of the knots is also important as explained by Rice [29], "...the key to the successful use of splines is to have the location of the knots as variables." To select the number and position of knots optimally, we propose using a linear least square method, called the subset regression technique [30]. Linear regression techniques may be used, because, in the spline function approximation, the nonlinearities are expressed as a sum of several functions, each multiplied by a different constant. In other words the nonlinearities are represented as linear functions of unknown parameters. Recently, the least squares method has been studied extensively by Mendel who developed efficient computation procedures for subset regression in both the batch and the sequential modes [31].

3.3.2 Estimation of Static Functional Relationships

Consider two scalar variables which are related by an unknown function

$$z = f(x) \qquad a \leq x \leq b \tag{3.40}$$

It is assumed that $f(x)$ is a continuous function in x over the closed interval $[a,b]$. Suppose that N pairs of values $(x(i), y(i))$, $a \leq x_i \leq b$ and $1 \leq i \leq N$ are given where $y(t)$ is a noise measurement of $z(i)$. The problem is to estimate a simple representation of the relationship between x and z from the data. Since the form of the nonlinear function is not known, it is approximated by a spline $S_{m,\nu}(x)$ with knots at $a = x_1 < x_2 < x_3 < \ldots x_{k+1} = b$. A general representation for such a spline is (see Schoenberg [32]):

$$S_{m,\nu}(x) = \sum_{j=0}^{m} C_{1j}\, x^j + \sum_{\ell=2}^{k} \sum_{j=\nu+1}^{m} C_{\ell j}\,(x-x_\ell)_+^j \qquad (3.41)$$

where x_+^j is the truncated power function

$$x_+^j = \begin{cases} x^j & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad (3.42)$$

The spline representation of Eq. (3.41) is very flexible. The knots can be taken in and out of the splines by adding new terms or removing certain terms. In addition, if $C_{\ell j}$ is zero for $j > m'$ and $\ell = 1,2,\ldots,k$, the spline $S_{m,\nu}(x)$ reduces to a lower order spline $S'_{m,\nu}(x)$. Thus, by selecting terms in Eq. (3.41) appropriately, a lower order spline or one with fewer knots may be obtained. The spline is a nonlinear function of the independent variable x, but is linear in the unknown parameters C. The linear least square regression methods are, therefore, applicable. When the least square method is used to select a subset of the terms from the equation, it is called the subset regression technique.

The basic problem in the successful use of splines for approximating any nonlinear function is the proper selection of:

(1) the order m and continuity ν of the splines, (b) number and position of knots, and (c) the various coefficients C_{ij}. We indicated above that the subset regression technique may be applied to drop terms out of the representation of Eq. (3.41) starting from the maximal representation (with most possible knots, highest possible order and lowest possible continuity ν). The method for selecting the more important terms, starting from the maximal representation, is now described.

Since $S_{m,\nu}(x)$ is used to approximate $f(x)$, Eq. (3.40) may be written as

$$z = \sum_{j=0}^{m} C_{1j} x^j + \sum_{\ell=2}^{k} \sum_{j=\nu+1}^{m} C_{\ell j}(x-x_\ell)_+^j + \varepsilon \qquad (3.43)$$

where ε is the modeling error because of approximating $f(x)$ by a spline function. Eq. (3.43) can be written for the given data points as:

$$y(i) = \sum_{j=0}^{m} C_{1j}(x(i))^j + \sum_{\ell=2}^{k} \sum_{j=\nu+1}^{m} C_{\ell j}(x(i) - x_\ell)_+^j$$

$$+ \varepsilon(i) , \qquad 1 \leq i \leq N \qquad (3.44)$$

where $\varepsilon(i)$ is the sum of modeling error and random measurement error. The problem is to select the fewest number of coefficients $C_{\ell j}$ which will provide an acceptable difference between $z(i)$ and $S_{m,\nu}(x(i))$ in the L^2 sense. Let,

$$y^T = [y(1), y(2), y(3), \ldots y(N)]$$

$$\theta^T = [C_{10}, C_{11}, \ldots, C_{1m}, C_{2(\nu+1)}, \ldots, C_{2m}, \ldots, C_{pm}]$$

$$\varepsilon^T = [\varepsilon(1), \varepsilon(2), \ldots \varepsilon(N)]$$

$$x_i = [1, x(i), x^2(i), \ldots, (x(i)-x_k)_+^m]$$

$$x = [x_1, x_2, \ldots, x_N] \qquad (3.45)$$

Equation (3.44) is written as

$$y = X\theta + \varepsilon$$

θ can be estimated by using the relation

$$\theta = (X^T X)^{-1} X^T y$$

however, a subset of the elements of the vector θ can explain most of the variation in y. The subset regression method selects this set of parameters iteratively. The procedure starts by including the variable most highly correlated with y. At every step F-tests are used to determine if a new variable may be included in the regression or if any of the variables already in the regression may be dropped. The cut-off F-value is determined from the F-distribution based upon a certain confidence level. If the cut-off value is higher than the partial F-value of any parameter outside the regression, that parameter may be included in the regression. On the other hand, if any parameter in the regression has a partial F-value smaller than cut-off value, that parameter may be dropped. The values of coefficients of the terms which are in the equation are computed simultaneously. For a detailed discussion of the method, see Efroymson [30].

The positions of the knots can be refined by using the following iterative procedure:

(1) Represent $f(x)$ by a sufficiently large number of knots at specific locations (e.g., distributed uniformly over the interval).

(2) Use subset regression to find which of these knots are important. Discard the remaining knots.

(3) Introduce several knots close to the knots selected in step (2).

(4) Repeat steps (2) and (3) until a good convergence occurs.

This procedure will give a good set of knots. These will be locally suboptimal to the extent the subset regression technique selects a suboptimal set of terms.

3.3.3 Representation of Nonlinear Systems Using Splines

The usefulness of an autoregressive moving average type representation in the input/output estimation of linear systems has been demonstrated previously. For the purpose of this section, we choose a form which resembles the autoregressive moving average form for the linear systems. The $p \times 1$ output vector $y(t)$ at time t is a function of M previous input and output vectors.

$$y(t) = \varphi(y(t-1),y(t-2),\ldots,y(t-M,u(t-1)$$

$$u(t-2),\ldots,u(t-M,t) + w(t) \qquad (3.48)$$

where φ is an unknown nonlinear function and $w(t)$ is a Gaussian random noise. The representation of Eq. (3.48) will be called N-ARMA. It is assumed that M is known. This representation has been used extensively in econometrics.

Using concepts introduced by Audley and Lee [24], it is quite clear that the unknown function φ cannot be estimated consistenly from finite data and from a single realization of the

process y. To get a useful representation of the nature of φ, it may be written as a spline function in terms of its arguments. Let $x(t)$ be the $n \times 1$ vector of the arguments of the function φ. Then, Eq. (3.48) may be written compactly as:

$$y(t) = \varphi(x(t)) + w(t) \tag{3.49}$$

The theory of general multivariable spline functions is not known because of several conceptual problems. One major problem is that splines over general multidimensional knot structures are not well-defined. In the present application, however, the knot structure may be chosen arbitrarily. It has been shown by Arthur [28] that a well-behaved polynomial spline may be defined over a multidimensional rectangular grid which will be called the net. To represent a multidimensional function $f(x)$, $x \; R^n$, in terms of splines over the rectangular nets (also called tensor product splines), the R^n space is divided into rectangles by lines

$$\begin{aligned} x_1 &= \xi_{11}, \xi_{12}, \cdots, \xi_{1p_1} \\ x_2 &= \xi_{21}, \xi_{22}, \cdots, \xi_{2p_2} \\ &\vdots \\ x_n &= \xi_{n1}, \xi_{n2}, \cdots, \xi_{np_n} \end{aligned} \tag{3.50}$$

A spline or order m in n variables, with continuous derivatives up to order $m-1$ can be written as

$$\begin{aligned} s_{m,m-1}(x) = & \sum_{j_1, j_2, \ldots, j_n = 0}^{m} \left\{ c_{j_1, j_2, \ldots, j_n} \prod_{i=1}^{n} x_i^{j_i} \right\} \\ & + \sum_{\ell_1=1}^{p_1} \sum_{\ell_2=1}^{p_2} \sum_{\ell_3=1}^{p_3} \cdots \sum_{\ell_n=1}^{p_n} \left\{ c^{\ell_1, \ell_2, \ldots \ell_n} \prod_{i=1}^{n} (x_i - \xi_{i\ell_i})_+^m \right\} \end{aligned} \tag{3.51}$$

This spline has a fixed polynomial representation in any of the multidimensional rectangles and has continuous derivatives up to order $(m-1)$ in each variable at the boundary of the rectangle. The definition can be extended when the spline has continuous derivatives of order $m-\nu$. For a general nonlinear system of Eq. (3.48), the above representation has many unknown constants even for small M and p_i. To obtain useful representation such that consistent estimation is possible, it is necessary to select the more important terms from the general spline representation.

The complexity in estimating φ depends upon the number of arguments in the unknown functions and the domain of each argument. Therefore, to simplify this problem, all known a priori information about the system must be used. The function φ must be reduced to the simplest form before attempting to approximate unknown nonlinearities. For example, the physics of the process may dictate that the function φ is a sum of nonlinear functions of the individual terms $y(t-i)$, $u(t-i)$, $i=1,2,\ldots,M$. Then, Eq. (3.48) simplifies to

$$y(t) = \sum_{i=1}^{M} \{\varphi_i(y(t-i)) + \psi_i(u(t-i))\} + w(t) \qquad (3.52)$$

All unknown nonlinear functions will be approximated by splines (e.g., in Eq. (3.52), each φ_i and ψ_i will be written as individual spline functions).

In nonlinear systems, defined by unknown functions, it is assumed that the behavior of the system in distant regions is significantly different. Therefore, from a given set of data, it is possible to estimate a representation of system behavior only over the region covered by the data. In contrast, the behavior of linear systems is the same everywhere; hence, a set of data provides the same information about its behavior in any part of the phase space. The rectangular net of Eq. (3.50) must take this into con-

sideration by not selecting any knots in regions where there is no data.

Starting from the general spline representation, we develop techniques for choosing important terms which describe the nonlinear behavior of the system in different regions. The next section illustrates the method for a first-order system with control nonlinearity. The general multivariable systems are discussed in Section 3.3.5.

3.3.4 Modeling of a Single Input, Single Output System With a Control Nonlinearity

To illustrate the technique, we discuss in this section, a system with only a single unknown function in one variable. Consider a single input, single output system in which the scalar output $y(t)$ follows the equation

$$y(t) = \sum_{\tau=1}^{M} a_\tau y(t-\tau) + \sum_{\tau=1}^{M} b_\tau g(u(t-\tau)) + w(t), \quad t > M \qquad (3.53)$$

The control input enters the system equation in a nonlinear manner. It is assumed that the function g does not depend upon the delay, τ, and the noise $w(t)$ is white. The problem is to find a good approximation for the nonlinear function $g(\cdot)$ given $y(s)$, $u(s)$, $s = 1,2,\ldots,N$.

The only unknown and nonlinear function in the above representation is $g(\cdot)$. Suppose that u varies between u_{min} and u_{max} for $s = 1,2,\ldots,N$. Then, from the given data, we may estimate $g(u)$ for $u_{min} < u < u_{max}$. The function $g(u)$ is written as the maximal spline function with knots ξ_i such that $u_{min} = \xi_1 < \xi_2 \cdots < \xi_{k+1} = u_{max}$.

$$g(u) \approx s_{m,\nu}(u) = \sum_{j=0}^{n} C_{1j} u^j + \sum_{\ell=2}^{k} \sum_{j=\nu+1}^{m} C_{\ell j} (u-\xi_\ell)_+^j \qquad (3.54)$$

The problem now is to select appropriate terms from the representation of Eq. (3.54) based upon the data. Substituting Eq. (3.54) in Eq. (3.53), and rearranging,

$$y(t) = \sum_{\tau=1}^{M} a_\tau y(t-\tau) + \sum_{j=0}^{m} C_{1j} \sum_{\tau=1}^{M} b_\tau (u(t-\tau))^j$$

$$+ \sum_{\ell=2}^{k} \sum_{j=\nu+1}^{m} C_{\ell j} \sum_{\tau=1}^{M} b_\tau (u(t-\tau) - \xi_\ell)_+^j + w(t) \qquad (3.55)$$

If a_τ and b_τ are known, Eq. (3.55) can be written as

$$y'(t) = \sum_{j=0}^{m} C_{1j} u_j(1,t) + \sum_{\ell=2}^{k} \sum_{j=\nu+1}^{m} C_{\ell j} u_j(\ell,t) + w(t) \qquad (3.56)$$

where

$$y'(t) = y(t) - \sum_{\tau=1}^{M} a_\tau y(t-\tau) \qquad (3.57)$$

$$u_j(\ell,t) = \begin{cases} \sum_{\tau=1}^{M} b_\tau (u(t-\tau))^j & \ell = 1 \\ \sum_{\tau=1}^{M} b_\tau (u(t-\tau) - \xi_\ell)_+^j & \ell \neq 1 \end{cases} \qquad (3.58)$$

Techniques of multiple regression can be applied to determine which coefficients C_{ij} should be included in Eq. (3.56) to represent the control nonlinearity.

If a_τ's are not known, an expression similar to Eq. (3.56) can be used to determine the important terms $C_{ij} \cdot y(t)$ is the first regressed on $y(t-\tau)$ followed by a multiple regression as described above. If b_τ's are not known, an iterative procedure must be applied. b_τ's are first estimated by assuming a certain form for the function $g(u)$, usually based upon a priori information (e.g., linear). These values are used in Eqs. (3.56) and (3.58) to determine important terms in the spline representation of the nonlinearity. This representation, in turn, is used to improve estimates of parameters b . This process is repeated until the convergence occurs. Though it has not been proven, this procedure seems to have good convergence properties in practical applications, as long as the initial representation for the function $g(u)$ is reasonable.

An example is now presented to demonstrate the technique.

Example 3.4

Consider a first-order system

$$x(t+1) = ax(t) + bg(u(t)) + w(t)$$

$$t = 1,2,\ldots 100$$

$$x(1) = 0 \qquad (3.59)$$

Let $a = 0.9$ and $b = 1.0$. g represents a nonlinearity of the saturation type

$$g(u) = \begin{cases} u & |u| \leq 1 \\ 1 & |u| \geq 1 \end{cases} \qquad (3.60)$$

The input u is

$$u(t) = \begin{cases} t/15 & 0 < t \leq 25 \\ 10/5 - t/15 & 25 < t \leq 75 \\ 5/15 - 20/3 & 75 < t \leq 100 \end{cases} \qquad (3.61)$$

u varies between -5/3 and 5/3. $g(u)$ is written as a spline function $S_{2,0}(u)$ with knots at -5/3, -4/3,...5/3, thus

$$g(u) \approx C_{10} + C_{11}u + C_{12}u^2 + \sum_{\ell=2}^{10} \sum_{j=1}^{2} C_{\ell j} \left(u+2 - \frac{\ell}{3}\right)_+^j \qquad (3.62)$$

Two cases are considered:

(1) $w(t)$ is white with standard deviation 1

(2) $w(t)$ is colored such that

$$w(t) = \varepsilon(t) + 0.5\varepsilon(t-1)$$

and $\varepsilon(t)$ is white with standard deviation 1.

For a 95 percent confidence level, the following expressions were obtained for the nonlinear function $g(u)$. (In case (b), $w(t)$ is assumed white during the multiple regression).

Case (1)

$$g(u) = \begin{cases} -1, & -5/3 \leq u \leq 1 \\ 1.04u + 0.04, & -1 < u \leq 1 \\ -0.13u + 1.21, & 1 \leq u \leq 5/3 \end{cases} \qquad (3.63)$$

Case (2)

$$g(u) = \begin{cases} -1, & -5/3 \leq u \leq -1 \\ 1.03u + 0.03, & -1 < u \leq 2/3 \\ -0.873u^2 + 2.19u - 0.358, & 2/3 < u \leq 5/3 \end{cases} \qquad (3.64)$$

The function $g(u)$ is shown in Figure 3.3. Although in case (b) the form of $g(u)$ is not correct, the actual value of $g(u)$ comes quite close to the estimated value. Note that out of a possible ten knots only two knots are included in each case. The assumption of whiteness of noise does not appear very critical. The methods for selection of knots and the order and continuity of spline functions discussed in Section II apply here.

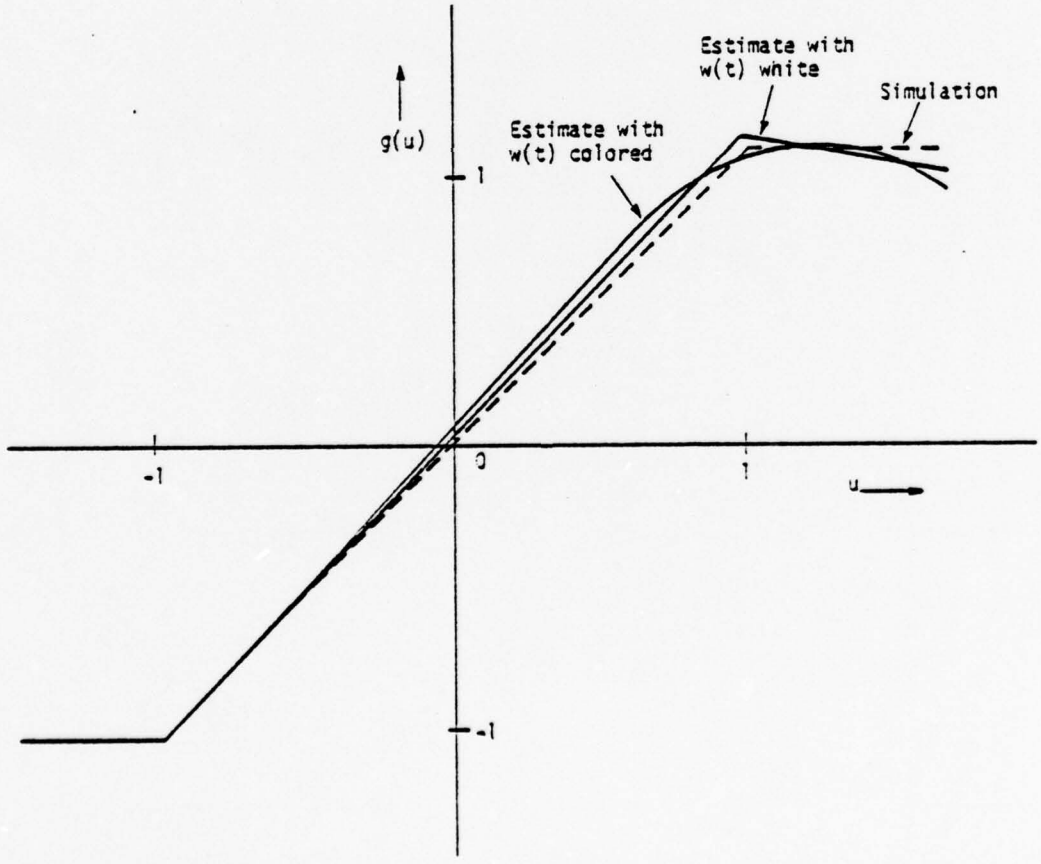

Figure 3.3 Plot of $g(u)$ as a Function of u (Simulated and Identified)

3.3.5 Multiinput/Multioutput Systems

The autoregressive moving average type of representation for a nonlinear system with unknown structure leads to Eq. (3.49). If the spline of Eq. (3.51) is used to approximate the nonlinearity, then Eq. (3.49) becomes

$$y(t) = S_{m,m-1}\,[x(t)] + w(t)$$

In theory, the technique described in the previous sections and those developed by Mendel [31] may be applied. The general problem, however, is quite complex. We describe here the application of spline representation to several special cases.

Unknown Time Varying System

A multiinput/multioutput time varying system with output y can be written in the ARMA form as

$$y(t) = \sum_{\tau=1}^{M} \{A_\tau(t)\, y(t-\tau) + B_\tau(t)\, u(t-\tau)\} + w(t) \qquad (3.66)$$

$A(t)$ and $B(t)$ are unknown functions of time. To find an approximation for these functions from measurements of $y(t)$ and $u(t)$, each of these functions is written as a spline in t.*

$$A_\tau(t) \approx S_{m,\nu}(t) = \sum_{j=0}^{m} C^\tau_{1j} t^j + \sum_{\ell=2}^{k} \sum_{j=\nu+1}^{m} C^\tau_{\ell j} (t-\tau_\ell)^j_+ \qquad (3.67)$$

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*Note that functions $A_\tau(t)$ and $B_\tau(t)$ are defined for integral t. These matrices possess certain continuity properties over these values for physical systems. Representing them in terms of continuous splines is, therefore, justified.

where $t_1, t_2, \ldots, t_k$ are the knots and C_{ij} are matrices of the same size as A. B is expanded in a similar fashion. Then the subset regression technique may be applied to obtain a useful spline approximation of each of the matrices.

Linearized Representation of a Nonlinear System

The above method can also be successfully applied to nonlinear systems to determine a linearized representation. For example, the general nonlinear system of Eq. (3.49) may be written as

$$dy(t) = \sum_{\tau=1}^{m} \left\{ \frac{\partial \varphi}{\partial y(t-\tau)} \, dy(t-\tau) + \frac{\partial \varphi}{\partial u(t-\tau)} \, du(t-\tau) \right\}$$

$$+ \text{Noise} \qquad (3.68)$$

Let,

$$\left. \frac{\partial \varphi}{\partial y(t-\tau)} \right|_t = A_\tau(t) \quad \text{and} \quad \left. \frac{\partial \varphi}{u(t-\tau)} \right|_t = B_\tau(t) \qquad (3.69)$$

Then,

$$dy(t) = \sum_{\tau=1}^{m} \{ A_\tau(t) \, dy(t-\tau) + B_\tau(t) \, du(t-\tau) \}$$

$$+\text{Noise} \qquad (3.70)$$

If A_τ and B_τ do not change too rapidly, the above equation can be approximated as

$$\Delta y(t) = \sum_{\tau=1}^{m} \{ A_\tau(t) \, \Delta y(t-\tau) + B_\tau(t) \, \Delta u(t-\tau) \}$$

$$+\text{Noise} \qquad (3.71)$$

$$\Delta y(t-\tau) = y(t-\tau+1) - y(t-\tau)$$

$$\Delta u(t-\tau) = u(t-\tau+1) - u(t-\tau) \qquad (3.72)$$

In the representation of Eq. (3.71), $A_\tau(t)$ and $B_\tau(t)$ can be determined as functions of t as described previously. The linearized representations of systems are useful in many applications (e.g., stability analysis, control design, etc.).

Systems Nonlinear in One Variable

Many systems are linear except some elements of the coefficient matrices depend upon a state, a control or a known function of some states, controls and time. The output y is written as

$$y(t) = \sum_{\tau=1}^{m} \{ A_\tau(\alpha)\, y(t-\tau) + B_\tau(\alpha)\, u(t-\tau) \} + w(t) \qquad (3.73)$$

where

$$\alpha = f(x(t), u(t), t) \qquad (3.74)$$

In this case also, the technique for time varying systems may be applied.

3.3.6 Application

The above procedure has been implemented using the multiple regression technique of Hall and Gupta [12]. It is applied to estimate the nonlinear aerodynamic behavior of a vehicle from flight data.

The simplified system has two states (pitch rate, q, and angle-of-attack, α) and two inputs (flap deflection, δ, and Mach number, μ). The equations of motion can be written as

$$\dot{\alpha} = Z(\alpha,\mu,\delta) + q$$

$$\dot{q} = M(\alpha,\mu,\delta) + M_q q \qquad (3.75)$$

There are noisy measurements of $\dot{\alpha},\alpha$, and q and the two controls. q is obtained by fitting a cubic spline to q. The problem is to determined $M_\alpha(t)$ and $Z_\alpha(t)$ for design of the control system. As described in Section V, Eq. (3.75) may be written as

$$\Delta\dot{\alpha} = Z_\alpha(t)\ \Delta\alpha + Z_\mu(t)\ \Delta u + Z_\delta(t)\ \Delta\delta + \Delta q$$
$$\Delta\dot{q} = M_\alpha(t)\ \Delta\alpha + M_\mu(t)\ \Delta u + M_\delta(t)\ \Delta\delta + M_q \Delta q \qquad (3.76)$$

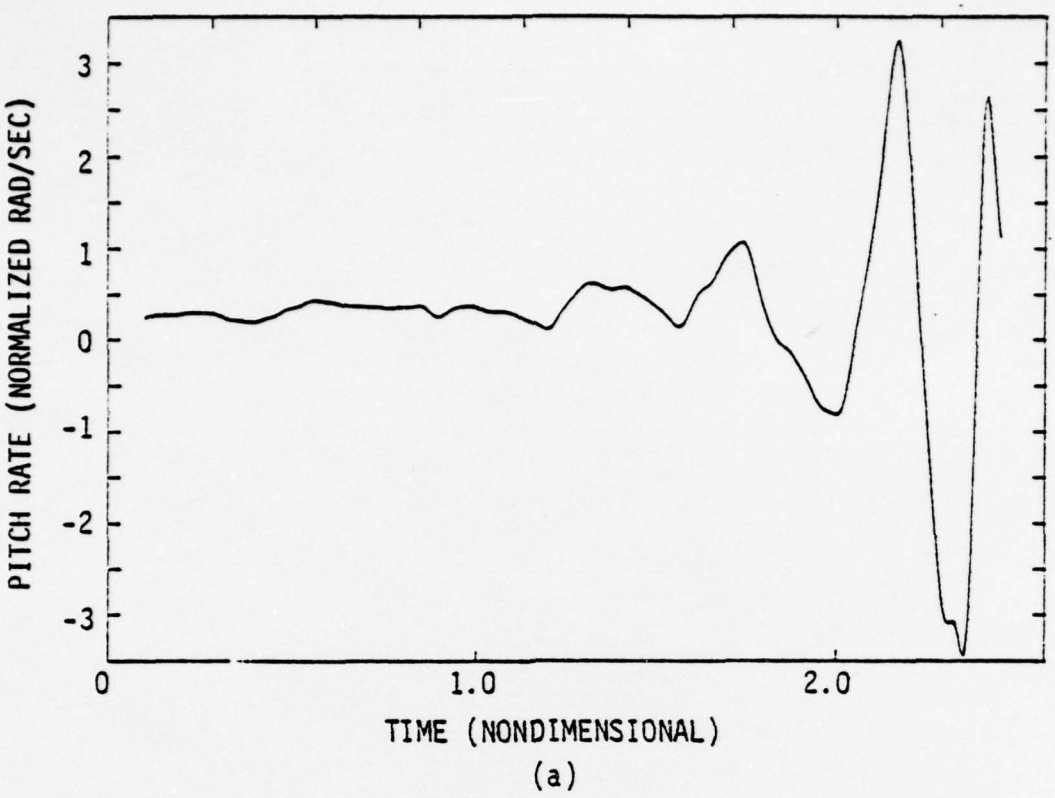
Time histories of α,μ,δ, and q are shown in Figure 3.4. $M_\alpha(t)$, $M_\mu(t)$ and $M_\delta(t)$ are written in terms of spline functions with knots equally spaced at 0.1 second intervals. The multiple regression technique is used to estimate the important terms in the spline representation. $M_\alpha(t)$ was identified to be:

$$M_\alpha(t) = -0.80 + 3.14(t-0.5)^2\ 0.2612(t-1.5)_+^2$$
$$266.2(t-1.9)_+^2\ -804.8(t-2.0)_+^2 + 1918(t-2.1)_+^2 \qquad (3.77)$$

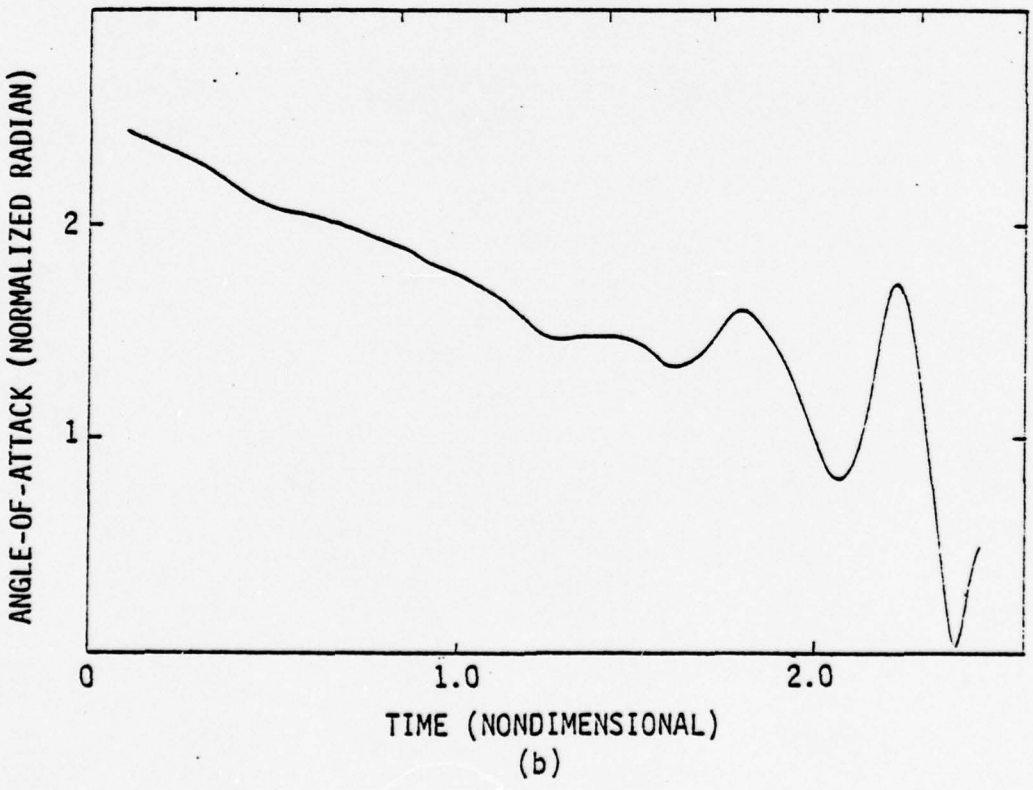
This function is plotted in Figure 3.5. The behavior of $M_\alpha(t)$ is quite close to what is expected from wind tunnel information.

3.3.7 Summary and Conclusions

This section discusses the application of polynomial splines in approximating unknown nonlinearities and time variations in dynamic systems. In the first step of the procedure, all unknown functional relationships are represented by general splines of high order and with many knots. Thereafter, the subset regression techniques are applied to select the more appropriate knots and

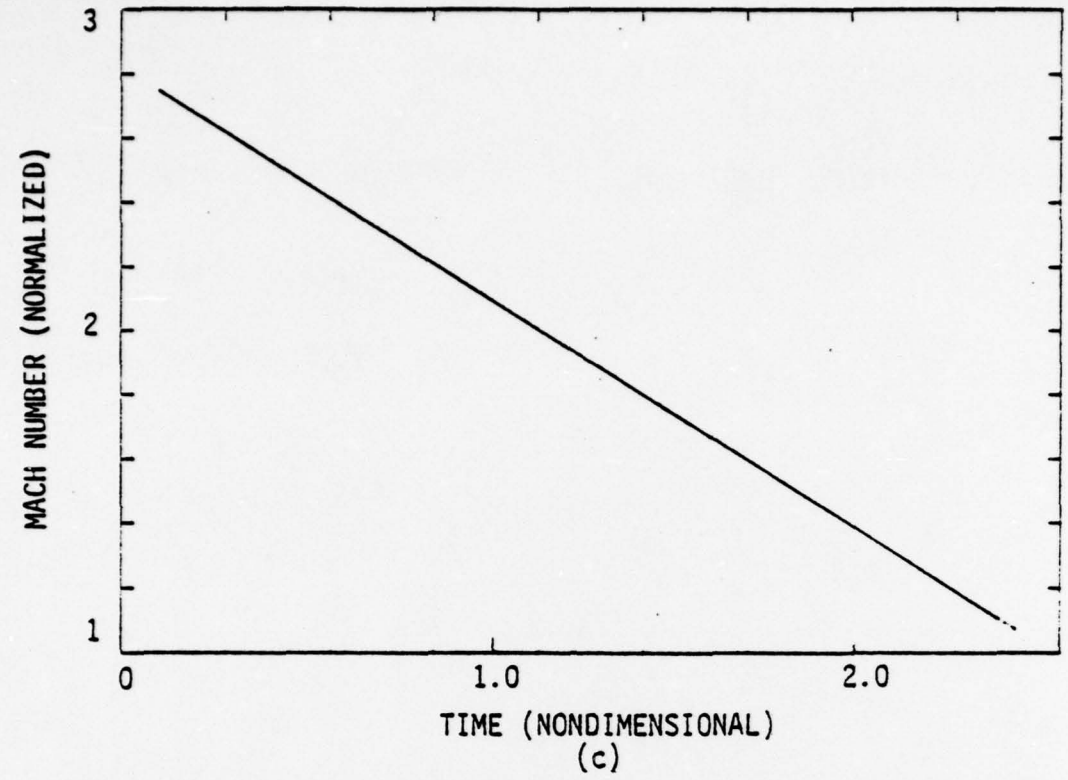


(a)

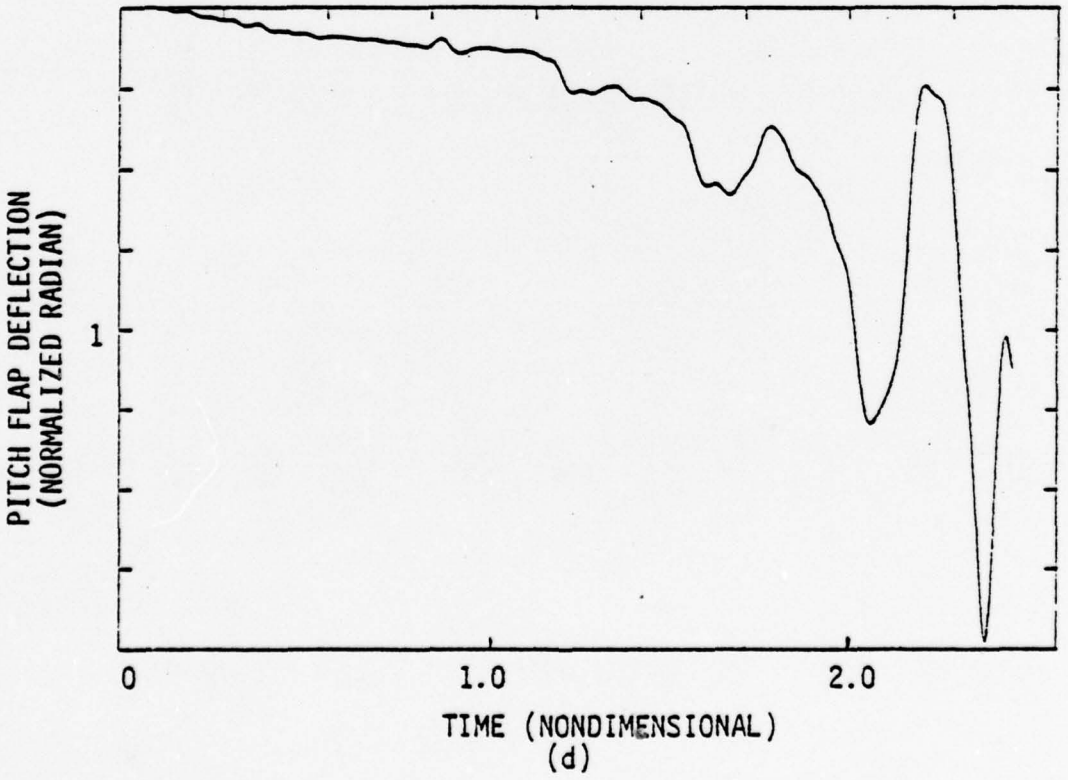


(b)

Figure 3.4 α, q, μ and δ Time Histories



(c)



(d)

Figure 3.4 (Continued)

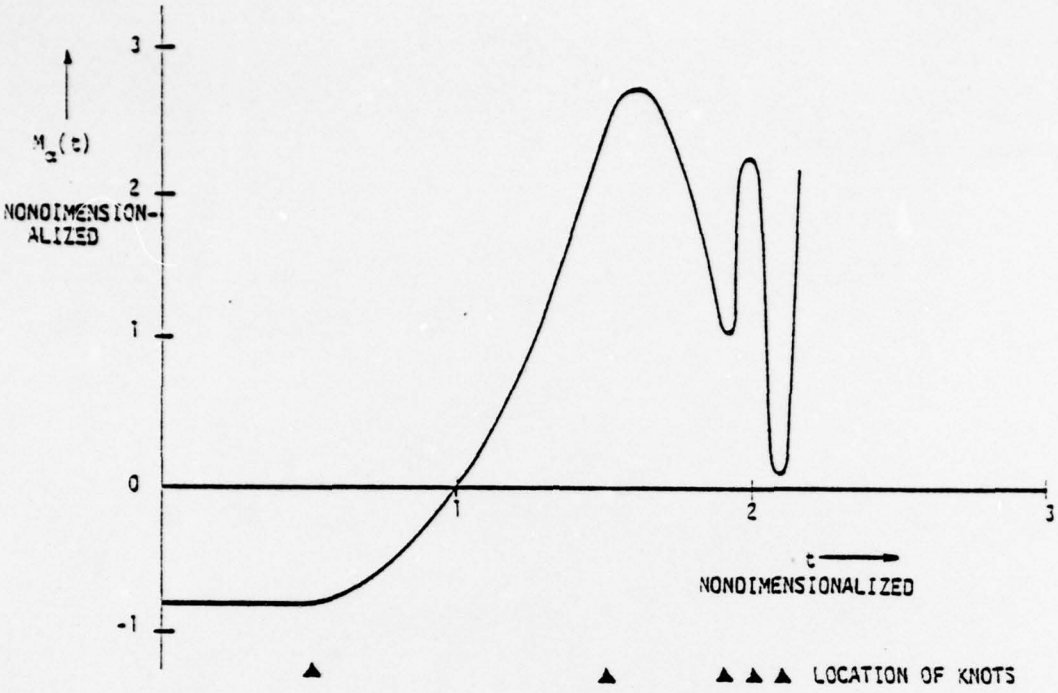


Figure 3.5 Variation of Identified $M_\alpha(t)$ with t

drop terms of high order if they are not important in explaining the observed response. This results in the best polynomial spline which approximates any nonlinearity. Techniques other than subset regression may be applied in selecting the more pertinent terms.

Two examples are presented to show the applicability of the above technique to SISO systems with simple control nonlinearities and to MIMO systems with multivariable nonlinearities. In both cases, excellent results are obtained even when the

assumption of white noise was violated. It is shown that the general polynomial splines of multivariable function over a rectangular net often involves too many terms. This stresses the importance of including a priori information about the structure of the system.

The spline functions are successful because of some of their excellent properties. Some of these are:

(1) The splines can closely approximate any single valued function in many variables.

(2) The behavior of a spline function in one region has very little influence on its behavior in another "distant" region.

(3) The knot structure of the fitted spline provides information on where greatest variation in a function is to be expected and, if this variation can be estimated.

(4) Spline representations often provide an insight into the physics of the system.

The above procedure is proposed to determine an approximate representation for the unknown nonlinear relationships. Final estimates of the various parameters in the spline representation will be done by using a more efficient and unbiased estimation method like the maximum likelihood and a computationally better conditioned form of the polynomial splines, called the B-splines [25].

IV. INPUT SELECTION TECHNIQUES FOR PARAMETER IDENTIFICATION AND MODEL DISCRIMINATION

The accuracy with which a useful model structure is isolated and values of various parameters are estimated depends upon three factors: (1) the input used to conduct the experiment, (2) the kind and quality of the measurements, and (3) the data processing algorithm. A proper selection of input is required for more efficient utilization of the test time and for obtaining more accurate results by processing a certain amount of data. In nonlinear problems, where the models are more complex and often unknown, this requirement becomes even more important. This has been demonstrated quite successfully by Hall and Gupta in Ref. 12.

The importance of choosing appropriate control inputs and exciting specific aircraft modes for extracting stability and control derivatives from aircraft flight testing has long been recognized. Good inputs could resolve parameter identifiability problems and improve confidences on estimates of stability and control derivatives obtained from the resulting flight test data. In other words, with specially chosen inputs the same accuracy on parameter estimates can be obtained in much shorter flight test time than with conventional inputs. Shorter flight tests can lead to a saving in time required for stability and control testing and the computation requirements for extraction of aerodynamic derivatives. In addition, these inputs can be chosen specifically to satisfy the ultimate flight test objectives such as control systems design, simulator parameter specifications, response prediction, aerodynamic model validation, or handling qualities evaluation. Previous work on the selection of parameter identifying inputs was discussed in Section II.

There are several methods of testing nonlinear systems. One method is to take several operating points and estimate an approximate linearized model around each operating points. This requires inputs which produce small perturbations around the operating point, such that the approximation of a linear model are valid. The techniques for designing such inputs are given in Section 4.1 and practical solution techniques are described in Section 4.2. The inputs which provide best model discrimination are described in Section 4.3. The summary and conclusions are described in Section 4.4.

4.1 INPUT DESIGN FOR PARAMETER IDENTIFICATION OF LINEARIZED MODELS AROUND AN OPERATING POINT -- THEORETICAL BACKGROUND

4.1.1 Problem Discussion

In linear operation regimes, the dynamic behavior of aerodynamic and hydrodynamic vehicles can be described by state space equations

$$\frac{d}{dt} x(t) = Fx(t) + Gu(t) \qquad 0 \leq t \leq T \tag{4.1}$$

x is the $n \times 1$ state vector (e.g., linear and angular position and rates) u is the $q \times 1$ input vector (e.g., hydrodynamic surface deflections) and F and G are matrices which represent aerodynamic and gravitational force coefficients and certain kinematic relationships. The vehicles are instrumented to measure certain response variables which are linear functionals of the state variables. The instruments have bias and random errors, therefore, the $p \times 1$ output y is written as

$$y(t) = Hx(t) + b + v(t) \tag{4.2}$$

b is the bias vector and $v(t)$ is a white Gaussian noise vector $v(t)$ has zero mean and power spectral density R. In general, H may also depend on some aerodynamic (or hydrodynamic) parameters. Examples of F, G and H are given in the next section.

Let α denote the vector of unknown aerodynamic parameters. It may be estimated from measurement of $y(t)$ and $u(t)$. The problem is to select an admissible input time history which provides the best estimates of the aerodynamic parameters based upon some criterion. Because of stresses and other considerations, the outputs are often constrained. Therefore, it is necessary to put an upper bound on a quadratic function of the state and the input, i.e.,

$$\int_0^T (x^T W_1 x + u^T W_2 u)\, dt \leq E \tag{4.3}$$

4.1.2 Mathematical Criteria for Input Design

A quantitative measure of the knowledge about a certain set of parameters in a given response is given by the information matrix. If M is the information matrix for the entire set of parameters (aerodynamic parameters α and unknown biases b), it was shown by Cramer and Rao [33] that

$$\mathrm{Cov}(\theta - \hat{\theta}) \geq M^{-1} \triangleq D \tag{4.4}$$

irrespective of the data reduction algorithm. θ is the true value (unknown) and $\hat{\theta}$ the estimated value of the parameters. In input design procedures discussed in the next section, it is assumed that an efficient identification algorithm is used such that all information about the parameter is extracted from data. The input design criterion can then be defined in terms of the information matrix. This assumption is important because it uncouples the input design procedure and the parameter identification algorithm.

It is usually not possible to find an input which gives better estimates of all aerodynamic parameters than any other input for a given operation condition of the vehicle. Therefore, scalar criteria have to be derived from the information matrix M. Since we are interested in the estimation of aerodynamic parameters only, it is necessary to consider parts of information and dispersion matrices corresponding to α. Call them $M^{(\alpha)}$ and $D^{(\alpha)}$, respectively. Clearly

$$M^{(\alpha)} = [I \mid 0]\, M \begin{bmatrix} I \\ \hline 0 \end{bmatrix}$$

$$D^{(\alpha)} = [I \mid 0]\, M^{-1} \begin{bmatrix} I \\ \hline 0 \end{bmatrix}$$

where I are $q \times q$ identity matrices.

Though several criteria based upon the information and dispersion matrices have been discussed in the literature [16], three of them are particularly useful for estimation of vehicle parameters:

(1) linear functional of $M^{(\alpha)}$

$$J_1 = \max_u \mathscr{L}(M^{(\alpha)}) \tag{4.5}$$

(2) the determinant of the dispersion matrix, $D^{(\alpha)}$

$$J_2 = \min_u |D^{(\alpha)}| \tag{4.6}$$

(3) linear functional of the dispersion matrix, $D^{(\alpha)}$

$$J_3 = \min_u \mathscr{L}(D^{(\alpha)}) \tag{4.7}$$

$\mathscr{L}$ is such that for two positive semi-definite matrices A and B and a constant c

$$(a)\quad \mathscr{L}(A) \geq 0$$

$$(b)\quad \mathscr{L}(A + B) = \mathscr{L}(A) + \mathscr{L}(B)$$

$$(c)\quad \mathscr{L}(cA) = c\mathscr{L}(A) \qquad (4.8)$$

Examples of linear operator $\mathscr{L}$ are the trace and the weighted trace.

J_1 maximizes the total or partial sum of information of all aerodynamic parameters or linear combinations of parameters. This may, however, lead to an almost singular information matrix implying a dispersion matrix with large diagonal terms. As such this criterion is unsuitable. It is useful because it arises as a subproblem in other cases.

The positive definite dispersion matrix can be considered to be the hyperellipsoid of uncertainty in the parameter space. J_2 works with the determinant of the dispersion matrix and minimizes the volume of the uncertainty ellipsoid. The advantage of this method is that it is independent of the units of the parameters and implies optimality of a prediction error criterion, see Mehra [17].

J_3 minimizes a weighted sum of covariances of parameter estimates (or some linear combinations of parameters). The weighting matrix serves two purposes. Since the covariances of different parameters have different units, it converts each term in the sum to the same units. Secondly, the weighting matrix offers tremendous flexibility because it is possible to assign varying importance to parameters, through weights on their nondimensional covariance. This is considered to be one of the most suitable performance criteria since it works with parameter estimate covariances directly.

4.1.3 Optimal Input Design

This section develops two techniques for input design: (1) the time domain approach, which gives a time history of the optimal input, and (2) the frequency domain approach, which gives the spectrum of the optimal input. We derive equations for the information matrix and then show algorithms to compute the optimal input.

4.1.3.1 Time Domain Approach

Information Matrix

The information matrix for parameters α and b, defining α as b element of the vector θ, is given by

$$M = \int_0^T \frac{\partial (Hx+b)^T}{\partial \theta} R^{-1} \frac{\partial (Hx+b)}{\partial \theta} dt \tag{4.9}$$

For simplicity, explicit time dependence is not shown where possible. Eq. (4.9) can be written as

$$M = \begin{bmatrix} M^{(\alpha)} & M^{(\alpha b)} \\ M^{(b\alpha)} & M^{(b)} \end{bmatrix} \tag{4.10}$$

where

$$M^{(\alpha)} = \int_0^T \frac{\partial (Hx)^T}{\partial \alpha} R^{-1} \frac{\partial (Hx)}{\partial \alpha} dt$$

$$M^{(\alpha b)} = \int_0^T \frac{\partial (Hx)^T}{\partial \alpha} R^{-1} dt$$

$$M^{(b)} = TR^{-1} \tag{4.11}$$

From Eq. (4.1)

$$x(t) = \int_0^t \varphi(t,\tau)\, Gu(\tau)\, d\tau \tag{4.12}$$

$\varphi(t,\tau)$, the transition matrix, obeys the following differential equations

$$\frac{\partial \varphi(t,\tau)}{\partial_t} = F\varphi(t,\tau) \qquad \varphi(\tau,\tau) = I \tag{4.13}$$

Using Eq. (4.12)

$$\frac{\partial (Hx)}{\partial \alpha} = \int_0^T \sum_{i=1}^{q} \frac{\partial}{\partial \alpha} \{ H\varphi(t,\tau) G_i \}\, u_i(\tau)\, d\tau \tag{4.14}$$

G_i is the ith column of G. Using Eqs. (4.11) and (4.14), it can be shown that

$$M^{(\alpha)} = \int_0^T \int_0^T \sum_{i,j=1}^{q} M_{ij}(\tau,s)\, u_i(\tau)\, u_j(s)\, d\tau ds$$

$$M^{(\alpha b)} = \int_0^T \sum_{i=1}^{q} A_i(\tau)\, u_i(\tau)\, d\tau \tag{4.15}$$

$M_{ij}(\tau,s)$ and $A_i(\tau)$ are derived in Ref. 16 and are:

$$M_{ij}(\tau,s) = \frac{1}{\delta} \int_{\sup(\tau,s)}^{T} \{A_i^T(t,\tau)R^{-1}(t)A_j(t,s)$$

$$+ A_j^T(t,\tau)R^{-1}(t)A_i(t,s)\}\, dt$$

$$A_i(t) = \int_o^T \frac{\partial}{\partial\alpha}\{ H\varphi(t,\tau)G_i\}\, dt$$

The constraint of Eq. (4.3) can be written as

$$\int_0^T\int_0^T u^T(\tau)\, Q(\tau,s)u(s)d\tau ds \leq E$$

$$q(\tau,s) = W_2\delta(\tau-s) + \int_{\sup(\tau,s)}^{T} G^T\varphi^T(t,\tau)W_1\varphi(t,s)Gdt \qquad (4.16)$$

where $\sup(\tau,\tau')$ is the larger of τ and τ' and δ is the Dirac delta function. Since $Q(\tau,s)$ is a positive semidefinite function,

the unequality sign in Eq. (4.16) can be converted into an equality sign. In addition, E may be taken as one. The optimal inputs can be scaled if the total energy is different from one. From Eqs. (4.10) and (4.15)

$$D^{(\alpha)^{-1}} = \int_0^T \int_0^T \sum_{i,j=1}^{q} \{ M_{ij}(\tau,s) - \frac{1}{T} A_i^T(\tau) R A_j(s) \} u_i(\tau) u_j(s) d\tau ds$$

which is a quadratic function of u, like $M^{(\alpha)}$. The methods which are described in this section are applicable as long as this quadratic relationship exists. For the sake of simplicity in this section, however, it is allowed that there is no bias error in the measurement. Then M and $M^{(\alpha)}$ are the same. From now on, M and $M^{(\alpha)}$ are used interchangeably.

Maximizing a Linear Functional of M

From Eqs. (4.5) and (4.15), the cost J_1 is

$$J_1 = \int_0^T \int_0^T \sum_{i,j=1}^{q} \mathscr{L}(M_{ij}(\tau,s)) u_i(\tau) u_j(s) d\tau ds$$

$$= \int_0^T \int_0^T u^T(\tau) P(\tau,s) u(s) d\tau ds \qquad (4.17)$$

$P(\tau,s)$ is a symmetric positive definite matrix. The cost function, as well as the constraint, is a quadratic function of the input. This optimization leads to the linear eigenvalue problem

$$\int_0^T P(\tau,\tau') u(\tau') d\tau' = \lambda \int_0^T Q(\tau,\tau') u(\tau') d\tau' \qquad (4.18)$$

The optimal input is the eigenvector corresponding to the maximum eigenvalue. Efficient computation methods to solve the equation are discussed in in the next section.

Minimization of the Determinant of D

The following result is proved in Appendix A.

Statement 4.1

A necessary and sufficient condition that an input $u^*(t)$ minimizes $|D|$ is that $u^*(t)$ is an eigenvector of the following equation corresponding to its maximum eigenvalue

$$\int_0^T P^*(\tau,s)\, u(s)ds = \lambda \int_0^T Q(\tau,s)\, u(s)ds$$

$$P_{ij}^*(\tau,s) = Tr(M^{*-1} M_{ij}(\tau,s)) \tag{4.19}$$

The maximum eigenvalue is m and '*' refers to the optimal input.

Eq. (4.19) cannot be solved directly because $P^*(\tau,s)$ is a function of the optimal input. This is a general nonlinear problem, which bears a close resemblance to Eq. (4.18), if the linear operator $\mathscr{L}$ in Eq. (4.17) is selected as

$$\mathscr{L}(\cdot) = Tr(M^{*-1}(\cdot)) \tag{4.20}$$

Fortunately, an iterative scheme which converges to the optimum input has been developed.

Algorithm 4.1

(1) Select an input $u_0(t)$ which satisfies the constraint of Eq. (4.16) and gives a nonsingular information matrix, M_0.

(2) Compute W based upon M_0.

(3) Compute the input $u_m(t)$ which maximizes the linear function of the information matrix defined by Eq. (4.20).

(4) Find a β_1 and β_2 such that $\sqrt{\beta_1}u_0(t) + \sqrt{\beta_2}u_m(t)$ gives the least $|D|$ under the constraint of Eq. (4.16) It is clear from the steps in the proof of Statement 4.1 that it is always possible to find β_1, β_2 which reduces the cost.

(5) Check if the input has converged. If not, return to step (b).

The proof of convergence of this algorithm is straightforward and is not given here.

Minimization of a Linear Functional of D

Statement 4.2

A necessary and sufficient condition that an input $u^*(t)$ minimizes $\mathscr{L}(D)$ is that $u^*(t)$ is an eigenvector of the following equation corresponding to its maximum value

$$\int_0^T P^*(\tau,s)u(s)ds = \lambda \int_0^T Q(\tau,s)u(s)ds$$

$$P_{ij}^*(\tau,s) = \mathscr{L}(M^{*-1}M_{ij}(\tau,s)M^{*-1}) \qquad (4.21)$$

The maximum eigenvalue is $\mathscr{L}(M^{*-1})$.

The proof is similar to the proof for Statement 4.1 and is not repeated. This again is a nonlinear problem and cannot be solved directly. Algorithm 4.1 can be used with the linear functional in step (c) defined by Eq. (4.21).

4.1.3.2 Frequency Domain Approach

If the duration of the test is long compared to the time constant of the system and the system is stable, frequency domain techniques can be used. In frequency domain, the optimal inputs can be computed more efficiently because: (1) the optimal input

has a discrete spectrum with finite frequencies [17], and (b) the response of a linear system at different frequencies is additive and independent. Instead of working with the time domain description of the system, the transfer function which describes the relation between the input and the output for different parameter values at all frequencies. For the system of Eqs. (4.1) and (4.2)

$$y(\omega) = H(j\omega I - F)^{-1} G\, u(\omega) + b\delta(\omega) + v(\omega) \tag{4.22}$$

If $T(\omega) \triangleq H(j\omega I - F)^{-1}G$, the various blocks of the information matrix per unit time are easily shown to be

$$M^{(\alpha)}_{ij} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{Tr}\, \{B_{ij}(\omega)\, dF_{uu}(\omega)\}$$

$$M^{(\alpha b)}_{i} = \frac{1}{2\pi} S_{uu}^{-1}(0) \frac{\partial T(0)}{\partial \alpha} u_s$$

$$M^{(b)} = \frac{1}{2\pi} S_{uu}^{-1}(0) \tag{4.23}$$

where

$$B_{ij}(\omega) = \frac{\partial T^*(\omega)}{\partial \alpha_i} S_{uu}^{-1}(\omega) \frac{\partial T}{\partial \alpha_i}(\omega) \tag{4.24}$$

$F_{uu}(\omega)$ is the spectral density of the input, '*' is complex conjugate and u_s is the constant part of the optimal input. It is clear that $M^{(\alpha b)}$ can be made zero by choosing u_s equal to zero (i.e., by making the optimal input to have zero power at zero frequency). Then the information matrix becomes

$$M = \begin{bmatrix} M^{(\alpha)} & 0 \\ 0 & M^{(b)} \end{bmatrix} \tag{4.25}$$

whose inverse is

$$M^{-1} = \begin{bmatrix} M^{(\alpha)^{-1}} & 0 \\ 0 & M^{(b)^{-1}} \end{bmatrix} \tag{4.26}$$

So the biases in measurements have no effect on $D^{(\alpha)}$. It is useful to effect this decoupling. Then $M^{(b)}$ can be discarded and M and $M^{(\alpha)}$ used interchangeably. In what is to follow this decoupling will be assumed and the optimal input will have zero power at zero frequency.

The constraint of Eq. (4.3), in frequency domain is

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Tr}(G^T(-j\omega I - F^T)^{-i} W_1 (j\omega I - F)^{-1} G + W_2)\, dF_{uu}(\omega) = E/T \tag{4.27}$$

or

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Tr}(A(\omega)\, dF_{uu}(\omega)) = E/T \tag{4.28}$$

The average power, E/T, may be taken to be one. The following result can be proved:

Statement 4.3

A necessary and sufficient condition that the input u with spectrum F^*_{uu} minimizes $|D|$ $(\mathrm{Tr}(WD))$ is that the maximum eigenvalue $\lambda(\omega)$ of the following equation be $m(\mathscr{Q}(D^*))$

$$B(\omega) = \lambda(\omega) A(\omega) \tag{4.29}$$

where

$$B(\omega) = \sum_{i,j=1}^{m} D^*_{ij} B_{ij}(\omega) \text{ to min } |D|$$

$$= \sum_{i,j=1}^{m} (WD^{*2})_{ij} B_{ij}(\omega) \text{ to min } Tr(WD) \qquad (4.30)$$

The proof of this statement is not given because it follows the proof of Statement 4.1 (described in Appendix A). The following algorithm can be used to compute the optimal input to minimize $|D|$ or $Tr(WD)$.

Algorithm 4.2

(1) Start with an input with spectrum F_0 such that the corresponding information matrix M_0 is nonsingular. Compute $A(\omega)$ and $B_{ij}(\omega)$ $(i,j=1,2,\ldots,m)$.

(2) Based upon the information matrix, compute $B(\omega)$.

(3) Find ω for which the biggest eigenvalue of

$$B(\omega) = \lambda(\omega)A(\omega) \qquad (4.31)$$

is maximum. Note that $A(\omega)$ and $B(\omega)$ are Hessenberg matrices, so that all the eigenvalues of the above equation are real. If λ_{max} is greater than m (if $|D|$ is to be minimized) or $Tr(WD)$ (if $TR(WD)$ is to be minimized) continue to the next step; otherwise, stop.

(4) Let $\psi^*_{max}(\omega)$ be the normalized eigenvector (viz. $\psi^*_{max}(\omega)\, A(\omega)\psi_{max}(\omega) = 1$) corresponding to the maximum eigenvalue λ_{max}. In general, $\psi_{max}(\omega)$ is complex. Update the design as follows

$$F_{k+1} = (1 - \alpha_k)F_k + \alpha_k \lambda_{max}(w)\ \psi^*_{max}(w) \qquad (4.32)$$

and choose α_k by a one-dimensional search. Return to step (b).

Most of the computations in the above algorithms are straightforward. Therefore, this algorithm requires much less computation time than the corresponding time domain algorithm. In step (c) the eigenvalue of a q x q matrix equation has to be determined at several values of ω. This research can usually be restricted to $(0,\omega_{max})$, where ω_{max} is related to the time constant of the system. In step (d), a Fibonacci search is applied because $|D|$ and $Tr(WD)$ are convex functions of α_k.

This algorithm specifies not only the optimal spectrum of each of the inputs, but also the various components of the cross-spectra. Since the optimal input has discrete spectra, it can be realized by a sum of sine waves. It can be approximated by using binary and other inputs. If the optimal input is implemented with a sum of sine waves, the cross-spectra terms give the phase relationship between the sine waves in various inputs.

4.1.4 A Suboptimal Time Domain Input

The computation of the optimal input in time domain is quite complex because it requires minimization over a function. A suboptimal input, which approximates the optimal input, can often be selected by the procedure described in this section. The suboptimal input is expressed as a linear combination of some basic input signals.

$$u(t) = \sum_{i=1}^{N} a_i \xi(i,t) \qquad (4.33)$$

$\xi_i(t)$ are some preselected function like steps, sines and cosines, etc. The coefficients a_i may be selected to obtain the best identifiability of parameters.

The constraints on the coefficients a_i because of the quadratic constraint on the input and the output may be written from Eq. (4.16) (with E assumed to equal one).

$$\int_0^T \int_0^T \sum_{i=1}^{N} a_i \xi^T(i,\tau) Q(\tau,s) \sum_{j=1}^{N} a_j \xi(j,s) d\tau \, ds = 1$$

or

$$A^T \bar{Q} a = 1 \tag{4.34}$$

where

$$\bar{Q}_{ij} = \int_0^T \int_0^T \xi^T(i,\tau) Q(\tau,s) \xi(j,s) d\tau \, ds$$

Similarly, the information matrix for the aerodynamic parameters is

$$M = \sum_{i,j=1}^{N} a_i a_j M^{ij} \tag{4.35}$$

where M^{ij} the cross information matrix defined as

$$M^{ij} = \int_0^T \int_0^T \sum_{k,\ell=1}^{q} M_{k\ell}(\tau,s) \xi_k(i,\tau) \xi_\ell(j,s) d\tau \, ds$$

The following algorithm may be used to minimize $|D|$, by selecting the coefficients a_i.

Algorithm 4.3

The following procedure converges to the global optimum:

(1) Compute and store Q and M^{ij} for i,j=1,2,...N .

(2) Select a vector a_0 such that the corresponding information matrix M_0 is nonsingular.

(3) Compute an N x N matrix P

$$P_{ij} = \mathrm{Tr}(M_0^{-1} M^{ij}) \tag{4.36}$$

(4) Find the largest eigenvalue λ_{max} and the corresponding eigenvector a_{max} of the equation

$$Pa = \lambda \bar{Q} a \tag{4.37}$$

If $\lambda_{max} = m$, stop; otherwise, continue to the next step.

(5) Update a by selecting β_1 and β_2 optimally,

$$a_{k+1} = \sqrt{\beta_1}\ a_k + \sqrt{\beta_2}\ a_{max} \tag{4.38}$$

such that the constraint of Eq. (4.34) is satisfied. Compute the new information matrix and return to (3).

Example 4.1

Consider a first order system with unknown input gain (e.g., model for roll control of a missile)

$$\dot{x} = -x + \theta u,\ x(0) = 0 \tag{4.39}$$

with noise measurements

$$y(t) = x(t) + v(t) \quad 0 \le t \le 2 \tag{4.40}$$

$v(t)$ is assumed to be white noise with unit power spectral density. An input is to be designed to get a good estimate of parameter θ. The optimal input is constrained.

$$\int_0^2 u^2(t)dt = 1 \tag{4.41}$$

(1) The optimal input for this system has been computed previously. It is

$$u(t) = 0.59\ (\sin 1.14t + 1.14 \cos 1.14t) \qquad (4.42)$$

The inverse of the information matrix (the covariance of parameter θ) is 2.29.

(2) A suboptimal input is designed; the input is chosen a sum of two functions

$$u(t) = a_1 s_1(t) + a_2 s_2(t-1) \qquad (4.43)$$

where $s_1(t)$ is a unit step function of unit duration at time $t=0$. The P and $\bar{Q}$ matrices are

$$\bar{Q} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.343 & 0.125 \\ 0.125 & 0.171 \end{bmatrix} \qquad (4.44)$$

the optimal input is easily shown to be

$$a_1 = 0.884$$

$$a_2 = 0.465$$

The inverse of the information matrix is 2.45. which is about 6.7 percent higher than the optimal input.

The optimal input and the suboptimal input are shown in Figure 4.1.

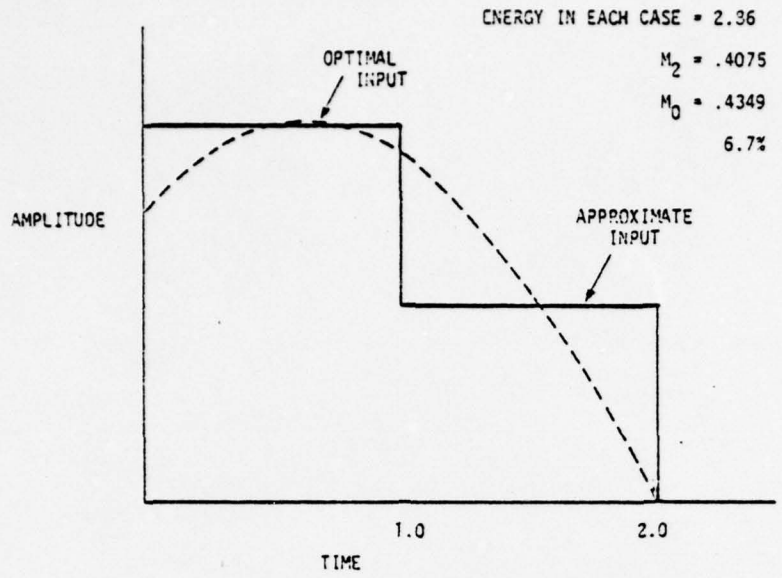


Figure 4.1 Optimal and Suboptimal Inputs for a First Order System

4.2 INPUT DESIGN FOR PARAMETER IDENTIFICATION OF LINEARIZED MODES AROUND AN OPERATING POINT - COMPUTATIONAL TECHNIQUES

4.2.1 Introduction

Computational methods have been a major hindrance in the routine computation and use of optimal inputs in aerodynamic flight testing. Efficient methods must be developed for systems with many state equations and a relatively large number of parameters. This part deals with the computational aspects of the parameter identifying inputs.

Iterative algorithms for minimizing the determinant or a linear functional of the dispersion matrix are given in the first part Section 4.1. The two major steps in the algorithms are the determination of an input which maximizes a linear function of the information matrix and the computation of the information matrix for various inputs. An efficient method for determining the information matrix has been described previously. This section

describes computational techniques for determining inputs which maximizes a linear function of the information matrix.

4.2.2 Numerical Methods for Maximizing a Weighted Trace of the Information Matrix

As before, let the state equations and the measurement equations be

$$\frac{d}{dt} x(t) = Fx(t) + Gu(t) \qquad 0 \leq t \leq T \tag{4.45}$$

$$y(t) = Hx(t) + b + v(t) \tag{4.46}$$

The information matrix for the aerodynamic parameters is

$$M = \int_0^T \frac{\partial (Hx)}{\partial \theta}^T R^{-1} \frac{\partial (Hx)}{\partial \theta} \tag{4.47}$$

Since

$$\frac{\partial (Hx)}{\partial \theta_i} = \frac{\partial H}{\partial \theta_i} x + H \frac{\partial x}{\partial \theta_i} \tag{4.48}$$

and

$$\frac{d}{dt} \frac{\partial x}{\partial \theta_i} = \frac{\partial F}{\partial \theta_i} x + F \frac{\partial x}{\partial \theta_i} + \frac{\partial G}{\partial \theta_i} u \tag{4.49}$$

If we define

$$x_\theta{}^T \triangleq \left(x^T, \frac{\partial x}{\partial \theta_i}^T, \ldots, \frac{\partial x}{\partial \theta_m}^T \right)$$

$$F_\theta \triangleq \begin{bmatrix} F & 0 & 0 & \cdots & 0 \\ \frac{\partial F}{\partial \theta_1} & F & 0 & \cdots & 0 \\ \frac{\partial F}{\partial \theta_2} & 0 & F & \cdots & 0 \\ \vdots & & & & \\ \frac{\partial F}{\partial \theta_m} & 0 & \cdots & \cdots & F \end{bmatrix}$$

$$G_\theta^T = \left(G^T, \frac{\partial G^T}{\partial \theta_1} \cdots \frac{\partial G^T}{\partial \theta_m}\right.$$

$$y_\theta^T = \left\{ (Hx)^T, \frac{\partial (Hx)^T}{\partial \theta_1} \cdots \frac{\partial (Hx)^T}{\partial \theta_m} \right\}$$

$$H_\theta \triangleq \begin{bmatrix} H & & \\ \frac{\partial H}{\partial \theta_1} & H & 0 \\ \vdots & \ddots & \\ \frac{\partial H}{\partial \theta_m} & \cdots & H \end{bmatrix} \qquad (4.50)$$

then

$$\dot{x}_\theta = F_\theta x_\theta + G_\theta u \qquad x_\theta(0) = 0$$

$$y_\theta = H_\theta x_\theta \qquad (4.51)$$

and the (i,j) element of the information matrix is

$$M_{ij} = \int_0^T y_{\theta i}^T R^{-1} y_{\theta j} dt \tag{4.52}$$

It has been shown in Ref. 34 that Eqs. (4.51) can be reduced to

$$\dot{x}_c = F_c x_c + G_c u$$

$$Y_\theta = T x_c \triangleq \left| \begin{array}{c} T_0 \\ - - \\ T_1 \\ - - \\ \vdots \\ - - \\ T_m \end{array} \right| x_c \tag{4.53}$$

Expressions for T, F_c and G_c are given in Ref. 2 and 34. From Eqs. (4.52) and (4.53)

$$M_{ij} = \int_0^T x_c^T(t)\, T_i^T R^{-1} T_j x_c(t) dt$$

$$= Tr\{ (T_i^T R^{-1} T_j) \int_0^T x_c^T(t) x_c(t) x_c(t) dt \} \tag{4.54}$$

Therefore,

$$Tr(WM) = \sum_{i,j=1}^{m} W_{ij} Tr\{ (T_i^T R^{-1} T_j) \int_0^T x_c^T(t) x_c(t) dt$$

$$= \int_0^T x_c^T(t) W^* x_c(t) dt \tag{4.55}$$

where

$$W^* = \sum_{i,j=1}^{m} W_{ij} T_i^T R^{-1} T_j$$

The quadratic constant on the input and the output may also be written in terms of $x_A(t)$ (with unit energy)

$$\int_0^T (x_c^T(t)T_0^T W_1 T_0 x_c(t) + u^T W_2 u)dt = 1 \qquad (4.56)$$

It can be shown, by an extension of the results of Ref. 16, that to maximize $Tr(WM)$, the following eigenvalue problem must be solved

$$\frac{d}{dt}\begin{bmatrix} x_c \\ \lambda \end{bmatrix} = \begin{bmatrix} F_c & -\mu G_c W_2^{-1} G_c^T \\ W^* - \mu T_0^T W_1 T_0 & -F_c \end{bmatrix} \begin{bmatrix} x_c \\ \lambda \end{bmatrix}$$

$$x_c(0) = \lambda(T) = 0 \qquad (4.57)$$

$$U_{opt} = -\mu W_2^{-1} G_c^T \lambda(t) \qquad (4.58)$$

It is assumed here that W_2 is nonsingular. If it is, we get singular control. For example, when W_2 is zero, Eq. (4.58) must be replaced by $G_c^T \lambda(t) = 0$. Again, we have the required number of equations to solve for all unknown qualities. The case of the singular control is not discussed.

The problem is to find the minimum μ such that Eq. (4.57) has a nontrivial solution because $1/\mu$ is the value of the corresponding performance index. The simplectic property of the Hamiltonian is used to find a solution to the eigenvalue problem. The eigenvalues of the state transition matrix occur in pairs $+\lambda$ and $-\lambda$. Let s_+ and s_- be the positive and negative sets of eigenvalues of this matrix with the corresponding partitioned eigenvector matrix

$$S = \begin{bmatrix} X_+ & X_- \\ \Lambda_+ & \Lambda_- \end{bmatrix} \tag{4.59}$$

normalized such that

$$\Lambda_-^T X_+ - X_-^T \Lambda_+ = I \tag{4.60}$$

It can be shown [16] that Eq. (4.57) has a nontrivial solution, if

$$U = \Lambda_+^{-1} \Lambda_- e^{-s_+ T} X_-^{-1} X_+ e^{-s_+ T} \tag{4.61}$$

has at least one eigenvalue equal to 1. A random search, followed, by a Newton-Raphson method, may be used to find the smallest μ such that an eigenvalue of this matrix is unity. This procedure is detailed in Ref. 16.

4.2.3 Numerical Computation of a Suboptimal Multistep Input

The algorithm makes it possible to compute the optimal inputs for high order systems with many unknown paramaters. The computation time may, however, be quite high. From a practical viewpoint a suboptimal, multi-step input may be sufficiently good. In this section, we develop an algorithm for selecting an input from a class of multi-step inputs. Suppose that an input with s steps each of duration Δ is to be selected. The optimal input may be written as

$$u(t) = \sum_{i=1}^{s} a_i \xi_i(t) \qquad 0 \leq t \leq T \tag{4.62}$$

where $\xi_i(t)$ is a unit step in the interval $(i-1)\Delta \leq t \leq i$. For sake of simplicity we assume that there is only one input. The multiinput case will be treated subsequently. Let $\bar{x}_c$ be the response of the reduced sensitivity equations to unput $\xi_1(t)$, (i.e.,

$$\dot{\bar{x}}_c = F_c \bar{x}_c + G_c \xi_1(t) \tag{4.63}$$

Then because of the linearity of the system and the fact that $\xi_2(t) \ldots \xi_s(t)$ are delayed $\xi_1(t)$, the state x_c for input u of Eq. (4.62) may be written as

$$x_c(t) = \sum_{i=1}^{s} a_i \bar{x}_c(t-(i-1)\Delta) \tag{4.64}$$

with the understanding that $\bar{x}(t) = 0$ for $t < 0$. From Eq. (4.55) weighted trace of the information matrix can be expressed in terms of a

$$\begin{aligned} Tr(WM) &= \int_0^T \sum_{i=1}^{s} a_i \bar{x}_c^T(t-(i-1)\ )W^* \sum_{j=1}^{s} a_j x_c(t-(j-1)s)dt \\ &= a^T P a \end{aligned} \tag{4.65}$$

W^* has been defined before and P is an $s \times s$ matrix such that

$$P_{ij} = \int_0^T \bar{x}_c^T(t-(i-1)\ )W^* \bar{x}_c(t-(j-1)\Delta)dt \tag{4.66}$$

P_{ij} can be computed by the following recursive relationship

$$P_{ij} = P_{i+1,j+1} + \int_0^{\Delta} \bar{x}_c^T(t+T-i\Delta)W^* x_c(t+T-j\Delta)dt$$

and

$$P_{i,s} = P_{s+1,i} = 0 \quad i,j=1,2,3,\ldots s \tag{4.67}$$

The quadratic constraint on the input and the output may also be expressed in terms of the variables a. From Eq. (4.56)

$$\int_0^T \sum_{i=1}^{s} \{ a_i \bar{x}_c^T(t-(i-1)\Delta) T_0^T W_1 T_0 \sum_{j=1}^{s} a_j \bar{x}_c(t-(j-1)\Delta)$$

$$+ \sum_{i=1}^{s} a_i \xi_i^T(t) W_2 \sum_{j=1}^{s} a_j \xi_j(t) \} dt = 1 \qquad (4.68)$$

i.e.,

$$a^T Q a = 1 \qquad (4.69)$$

where

$$Q_{ij} = \int_0^T \{ \bar{x}_c^T(t-(i-1)\Delta) T_0^T W_o T_0 \bar{x}_c(t-(j-1)\Delta)$$

$$+ \xi_i^T(t) W_2 \xi_j(t) \} dt \qquad (4.70)$$

Note that both P and Q are written in terms of the single time history propagation of the reduced sensitivity function. This makes it possible to compute the suboptimal multi-step input using the following steps (this is a special application of Algorithm 4.3, Section 4.1).

Steps in the Computation of the Suboptimal Multi-Step Input

(1) Using the state equations and the unknown parameters from the reduced sensitivity functions Eq. (4.53), (i.e., compute F_c, G_c and T).

(2) Select the number of steps s and propagate Eq. (4.63) and store $\bar{x}_c$.

(3) Compute and store Q using Eq. (4.70).

(4) Select an $s \times 1$ vector a_0 and compute $x_c(t)$ from Eq. (4.64).

(5) Find the information matrix M_0.

(6) Calculate W based upon the criterion for optimization and M_0. Also compute W^*.

(7) Compute P using Eq. (4.67).

(8) Determine the highest eigenvalue λ_{max} and the corresponding eigenvector a_{max} of the following

$$P_a = \lambda Q a \tag{4.71}$$

(9) Update the design

$$a_{i+1} = \sqrt{\beta_1} a_i + \sqrt{\beta_2} a_{max} \tag{4.72}$$

such that a_{i+1} satisfies Eq. (4.70) and optimizes the criterion function. If the criterion function improves very little, stop; otherwise return to step (5).

4.3 INPUT DESIGN FOR MODEL DISCRIMINATION

Sometimes, in complex processes, several competing models can be postulated. Tests may then be conducted to discriminate among these models. In linear systems this may involve selecting the order of the process and various canonical indices. This problem is more complex in nonlinear systems where the natures of the nonlinearities may have to be discriminated for, in addition to determining the order of the process. The inputs used to excite the system during the test may resolve the model ambiguity with lower chance of error and in a shorter test time.

4.3.1 Problem Statement

A process may follow one of several models, 1,2,...N. The ith model may be described by

$$\dot{x}_i = f(x_i, u, \theta_i) + w_i, \quad x(0) = 0 \qquad 0 \leq t \leq T \tag{4.73}$$

with measurements,

$$y_i = h_i(x_i,u,\theta_i) + v_i$$

x_i is the state vector for the ith model and θ_i is a set of unknown parameters in the model. w_i and v_i are random white noise sources with power spectral densitites of Q and R, respectively (known or unknown). The problem is to determine the most likely model given a certain input u and the corresponding response y.

For the sake of simplicity, we assume that there are two competing models. The case of multiple models is a straightforward extension.

4.3.2 The Criterion Function

Let us suppose that the two dynamic models are

Model 4.1

$$\dot{x}_1 = f_1(x_1,u,\theta_1)$$

$$y_1 = h_1(x_1,u,\theta_1) + v_1 \qquad (4.74)$$

Model 4.2

$$\dot{x}_2 = f_2(x_2,u,\theta_2)$$

$$y_2 = h_2(x_2,u,\theta_2) + v_2 \qquad (4.75)$$

Note that the two state vectors may be of different dimensions and the number of unknown parameters in the two models may be different. We have assumed that the process noise does not exist, because the extension to the process noise case may be obtained directly by using the innovations representation.

Suppose an input u is applied to the system and an output y observed. The likelihood that the first model is the true model is

$$S_1 = p(y_1=y/u,\theta_1,M_1) \tag{4.76}$$

The negative logarithm of this likelihood function is

$$L_1 = \frac{1}{2}\int_0^T \{ (y-h_1(x_1,u,\theta_1))^T R_1^{-1}(y-h_1(x_1,u,\theta_1)) + \log |R_1| \} dt \tag{4.77}$$

The negative log-likelihood function of the second model being the true model is also computed in a similar fashion. We begin with the assumption that there are no unknown parameters in either model. Then, if M_1 is the true model, the expected values of L_1 and L_2 are

$$E(L_1(M_1)) = E\,\frac{1}{2}\int_0^T \{ (h_1+v_1-h_1)^T R_1^{-1}(h_1+v_1-h_1) + \log |R_1| \} dt$$

$$= \frac{1}{2}(p + \log |R_1|)T \tag{4.78}$$

$$E(L_2(M_1)) = E\,\frac{1}{2}\int_0^T \{ (h_2+v_1-h_2)^T R_2^{-1}(h_1+v_1-h_2) + \log |R_1| \} dt$$

$$= \frac{1}{2}(Tr(R_1R_2^{-1}) + \log |R_2|)T + \frac{1}{2}\int_0^T (h_1-h_2)^T R_2^{-1}(h_1-h_2)dt \tag{4.79}$$

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Using [8]

$$E(L_1(M_1)) = \frac{1}{2}(p + \log|R_1|)T - \frac{1}{2}m_1 \tag{4.84}$$

where m_1 is the number of unknown parameters in the first model. Computation of $E(L_2(M_1))$ is more difficult. Again the estimated value of θ_2 follows the equation

$$\int_0^T (h_1+v_1-h_2)R_2^{-1}\frac{\partial h_2}{\partial \theta_2}dt = 0 \tag{4.85}$$

Note that there could be a large error in the estimated value of θ_2 because an incorrect model is used in the identification. Let θ_{21} be the estimated value of θ_2 if there is no noise in the measurement. Assuming θ_2 is close to θ_{21} it is easy to show that

$$E(L_2(M_1)) = \frac{1}{2}(Tr(R_1R_2^{-1}) + \log|R_2|)T - \frac{1}{2}m_2$$

$$+ \frac{1}{2}\int_0^T (h_1(\theta_1) - h_2(\theta_{21}))^T R_2^{-1}(h_1(\theta_1)$$

$$- h_2(\theta_{21}))dt \tag{4.86}$$

$_{21}$ is a parameter vector such that $h_2(\theta_{21})$ is the projection of $h_2(\theta_2)$ on $h_1(\theta_1)$. If we take the difference of $E(L_1(M_1))$. A criterion similar to that of Eq. (4.80) will be obtained. Again, we must maximize the last term in the above equation. Note that θ_{21} depends upon the applied input.

4.3.3 Optimal Input Selection

The above discussion shows that an input which maximizes J must be selected.

$$J = \frac{1}{2}\int_0^T ||h_1(x_1,u,\theta_1) - h_2(x_2,u,\theta_{21})||_{R_2^{-1}} dt \qquad (4.87)$$

where θ_{21} is given by

$$\int_0^T (h_1(x_1,u,\theta_1) - h_2(x_2,u,\theta_{21}))^T R_2^{-1} \frac{\partial h_2}{\partial \theta_{21}} (x_2,u,\theta_{21})\, dt = 0 \qquad (4.88)$$

where x_1 and x_2 follow the equations

$$\dot{x}_1 = f_1(x_1,u,\theta_1), \quad x_1(0) = 0 \qquad (4.89)$$

and

$$\dot{x}_2 = f_2(x_2,u,\theta_{21}) \quad x_2(0) = 0 \qquad (4.90)$$

The constraints of the equation can be removed by selecting an m_2 state variable vector x_3.

$$\dot{x}_3 = \frac{\partial h_2^T}{\partial \theta_{21}} (x_{21},u,\theta_{21}) R_2^{-1} (h_1(x_1,u,\theta_1) - h_2(x_{21}u,\theta_{21}))$$

$$x_3(0) = x_3(T) = 0 \qquad (4.91)$$

Let us assume that u must belong to a set of real function R. Then, the problem is to

$$\min_{u \in R, \theta_{21}} J = \frac{1}{2} \int_0^T ||h_1(x_1,u,\theta_1) - h_2(x_2,u,\theta_{21})||_{R_2^{-1}} dt \qquad (4.92)$$

Subject to the constraints of Eq. (4.89), (4.90) and (4.91).

Note that θ_{21} is a vector of constant controls. This is a standard problem in control theory.

Example 4.2

$$\text{M1:} \quad \dot{x}_1 = -x_1 + u$$

$$y_1 = x_1 + v \qquad 0 \leq t \leq 2 \qquad (4.93)$$

$$\text{M2:} \quad \dot{x}_2 = -2x_2 + 2u$$

$$y_2 = x_2 + v \qquad 0 \leq t \leq 2 \qquad (4.94)$$

We wish to select an input consisting of two steps each of one second duration to discriminate between these two models. Let the steps be of amplitude a_1 and a_2. Then it is straightforward to show that

$$x_1 = a_1(1-e^{-t}) \qquad 0 \leq t \leq 1$$

$$= a_1(e-1)e^{-t} + a_2(1-ee^{-t}) \qquad 1 \leq t \leq 2$$

$$x_2 = a_1(1-e^{-2t}) \qquad 0 \leq t \leq 1$$

$$= a_1(e^2-1)e^{-2t} + a_2(1-e^2e^{-2t}) \qquad 1 \leq t \leq 2$$

A straightforward computation gives

$$\int_0^2 (x_1-x_2)^2 \, dt = 0.5428a_1^2 - 0.0228a_1a_2 + 0.04428a_2^2$$

If there is a total energy constraint on the input the optimal input is such that

$$\frac{a_1}{a_2} = -1.53$$

4.4 CONCLUSIONS

This chapter presented various techniques for designing tests which make best use of the experiment time in terms of obtaining the most accurate parameter estimates within the constraints of the system. Various methodologies for conducting tests on nonlinear systems are discussed first. This includes direct estimation of the nonlinear relationships and of conducting tests at several operating points and estimating the small signal (linearized) model at each point. Input design techniques for both cases are covered. A special emphasis is given to the determination of inputs which are suboptimal, but easy to compute.

In nonlinear systems, we often need model discriminating inputs. Methods for the selection of such inputs are covered in detail.

V. CONCLUSIONS

To extend the application of system identification technology to complex Navy systems, two major problems have been solved. These are: (a) the development of general techniques to determine the structure of a nonlinear model which best explains an input/output data, and (b) the development of methods for designing inputs and maneuvers which adequately excite each mode to provide accurate estimates of all unknown parameters in the identification stage.

An effective model structure determination technique is developed as a combination of two methods

(1) Selection of parameterization: This method selects a set of important parameters from a general model. The parameters may correspond to coupling between various parts of the model or the order of dynamics in the model.

(2) Spline representation of nonlinearities: This method uses a general spline representation for unknown nonlinearities and then applies optimal subset regression to select the appropriate terms from the general spline representation.

The input design technique for nonlinear systems is a combination of the following methods.

(1) Inputs for small motions around operating points: Several operating points are selected to cover the nonlinear operating region. Inputs are designed for small motions around these operating points.

(2) Computational techniques using orthogonal functions: Computational techniques, based on representing the input as a sum of orthogonal functions, are developed to enable determination of optimal inputs for complex nonlinear systems.

(3) Optimal inputs for model discrimination: Inputs, which enhance the distinguishability among several competing models, are designed.

These methods have been used with simulation data and are currently being implemented for flight test data application.

The results of the present study lead to the following major conclusions:

(1) Model structure determination is an important part of the system identification problem in complex nonlinear systems. Techniques for model structure determination from noisy data have been developed and are computationally feasible.

(2) Inputs must be carefully selected to be able to estimate the model structure and the parameters accurately. Techniques for designing inputs to improve parameter estimation accuracy or model distinguishability have been developed.

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